

Mineral Resource Classification

"Mineral Resources vs Reserves, Classification Systems, Measured / Indicated / Inferred Concepts, Economic Considerations"

1. Basic Definitions

➔ Mineral Resource:

A concentration or occurrence of mineral material of economic interest in or on the Earth's crust with reasonable prospects for eventual economic extraction.

➔ Mineral Reserve:

The economically mineable part of a Measured or Indicated Resource demonstrated by at least a Preliminary Feasibility Study.

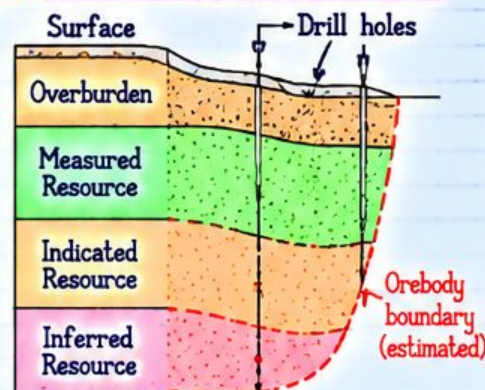
2. Classification Systems

Common classification systems:

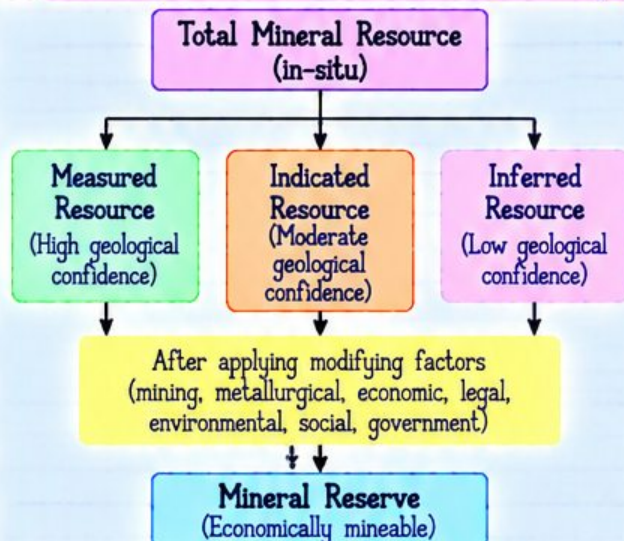
- ✓ UNFC - United Nations Framework Classification
- ✓ JORC Code - Australia
- ✓ NI 43-101 - Canada
- ✓ CRIRSCO - International reporting standard
- ✓ USGS classification

All systems classify mineral quantities based on geological confidence and economic viability.

3. Schematic Representation



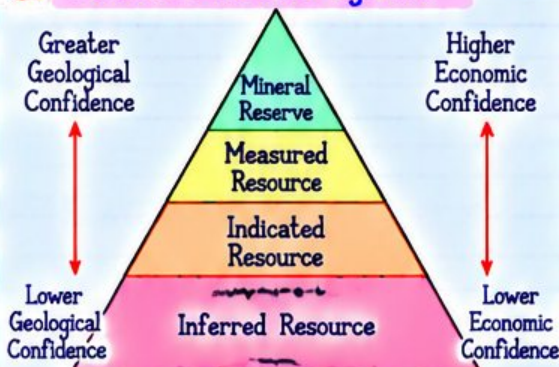
4. Mineral Resource - Reserve Relationship



5. Key Characteristics

Category	Geological Confidence	Data Spacing (typical)	Key Features
Measured Resource	High	Close spacing (e.g., 25-50 m)	- Well defined geometry - Reliable grade estimate - Continuity established
Indicated Resource	Moderate	Medium spacing (e.g., 50-200 m)	- Reasonable continuity - Acceptable grade estimate - Basis for mine planning
Inferred Resource	Low	Wide spacing (e.g., 200-1000 m)	- Limited geological evidence - Uncertain grade and continuity - Used for early stage evaluation
Mineral Reserve	Economic certainty	Based on feasibility study	- Technically and economically mineable part of Measured/Indicated Resource - Includes modifying factors

6. Resource-Reserve Pyramid



7. Economic Considerations

- ✓ Grade and quality of mineral.
- ✓ Size and shape of the deposit.
- ✓ Mining and processing methods.
- ✓ Metallurgical recoveries.
- ✓ Market demand and price of mineral.
- ✓ Operating and capital costs.
- ✓ Cut-off grade.
- ✓ Legal, environmental and social factors.
- ✓ Infrastructure and location.
- ✓ Government policies and taxation.
- ✓ Risk and uncertainty analysis.
- ✓ Life of mine and profitability (NPV, IRR).

8. Important Points for GATE

- ➔ Resources are geological in nature.
- ➔ Reserves are a subset of resources and are economically mineable.
- ➔ Measured > Indicated > Inferred in terms of geological confidence.
- ➔ Only Measured and Indicated resources can be converted to reserves.
- ➔ Inferred resources cannot be directly converted to reserves.
- ➔ Modifying factors include mining, metallurgical, economic, legal, environmental, social and government considerations.
- ➔ Classification helps in mine planning, valuation and resource management.

Key Takeaways

- ✓ Resource = Geological quantity.
- ✓ Reserve = Economically mineable part.
- ✓ Measured, Indicated, Inferred based on confidence.
- ✓ Reserves are derived from resources after applying modifying factors.
- ✓ Classification systems: UNFC, JORC, NI 43-101.

Common Modifying Factors

- ✓ Mining method
- ✓ Metallurgical recovery
- ✓ Economic analysis (price, cost, cut-off grade)
- ✓ Legal and environmental factors
- ✓ Infrastructure and location
- ✓ Government policies and taxation

"Right Classification
Today,
Better Planning
Tomorrow,
Sustainable Mining
for the Future."

Discounted Cash Flow Analysis

"Time Value of Money, Cash Flow, Discount Rate, NPV, IRR, Payback Period"

1. Time Value of Money

- A rupee today is worth more than a rupee tomorrow due to its earning potential.

Future Value (FV):
 $FV = PV (1 + i)^n$

Present Value (PV):
 $PV = \frac{FV}{(1 + i)^n}$



PV = Present Value (₹)

FV = Future Value (₹)

i = interest/discount rate (per year)

n = number of years

4. Net Present Value (NPV)

- NPV is the sum of present values of all cash inflows and outflows.

$$NPV = \sum_{t=0}^n \frac{CF_t}{(1 + i)^t}$$

CF_t = Cash flow at year t

i = Discount rate

n = Project life (years)



Criteria:

- ✓ NPV > 0 → Accept project
- ✓ NPV = 0 → Break-even
- ✓ NPV < 0 → Reject project

7. Example - NPV Calculation

A mine project has the following cash flows. Discount rate = 10%. Find NPV.

Year (t)	Cash Flow (₹ crore)
0	-500
1	150
2	200
3	250
4	200
5	150

$$NPV = \sum_{t=0}^5 \frac{CF_t}{(1 + 0.10)^t}$$

NPV = + ₹ 191.74 crore
(Positive → Accept)

2. Cash Flow in Mining Projects

- Initial Investment (Year 0):**
Land, exploration, mine development, plant & machinery, infrastructure.
- Operating Cash Inflows:**
Revenue from sale of minerals.
- Operating Cash Outflows:**
Mining, processing, logistics, labour, power, maintenance, administration, taxes.
- Terminal Value:**
Salvage value at the end of mine life.

Types of Cash Flow:

- ✓ Pre-tax cash flow
- ✓ After-tax cash flow (more realistic for evaluation)



5. Internal Rate of Return (IRR)

- IRR is the discount rate at which NPV = 0.

$$0 = \sum_{t=0}^n \frac{CF_t}{(1 + IRR)^t}$$

Criteria:

- ✓ IRR > Required return → Accept
- ✓ IRR = Required return → Indifferent
- ✓ IRR < Required return → Reject

Notes:

- ✓ Use trial and error or financial calculator or spreadsheet to find IRR.
- ✓ For non-conventional cash flows, multiple IRRs may exist.

8. Example - IRR Calculation

Find IRR for the following cash flows.

Year (t)	Cash Flow (₹ crore)
0	-400
1	100
2	150
3	200
4	150

Using trial and error:

$$0 = \sum_{t=0}^4 \frac{CF_t}{(1 + IRR)^t}$$

IRR ≈ 18.6%
(If required return = 12% → Accept)



3. Discount Rate (i)

- The rate used to convert future cash flows to present value.
- Reflects risk, cost of capital and opportunity cost.

Factors affecting discount rate:

- ✓ Project risk
- ✓ Country/political risk
- ✓ Inflation
- ✓ Cost of capital (WACC)
- ✓ Industry risk



Typical Range for Mining Projects:

- ✓ 8% - 12% (stable projects)
- ✓ 12% - 18% (high risk projects)

6. Payback Period

- Time required to recover the initial investment from cumulative cash inflows.

$$\text{Payback Period} = \frac{\text{Initial Investment}}{\text{Annual Net Cash Inflow}}$$

Types:

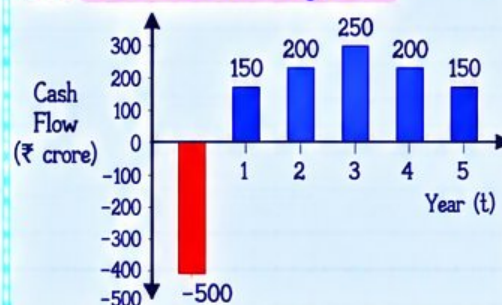
- ✓ Simple Payback Period (ignores time value)
- ✓ Discounted Payback Period (considers time value of money)



Criteria:

- ✓ Shorter payback period is better.
- ✓ Commonly used for quick assessment, but not sufficient alone.

9. Cash Flow Diagram



10. Key Takeaways

- ✓ Understand time value of money.
- ✓ Be comfortable with NPV, IRR and Payback.
- ✓ Use cash flow diagrams for clarity.
- ✓ Consider realistic discount rate.
- ✓ Use after-tax cash flows for better decision.
- ✓ Compare multiple projects using NPV and IRR.
- ✓ Solve GATE-level numerical problems.



Mine Valuation

"Principles, Costs, Revenue, NPV, Profitability Indicators & Numericals"

1. Mine Valuation - Concept

➔ Mine valuation is the process of estimating the economic worth of a mine based on its mineral reserves, expected cash flows and associated risks.

➔ Used for:

- ✓ Project feasibility
- ✓ Investment decisions
- ✓ Mergers and acquisitions
- ✓ Loan and financing
- ✓ Divestment / sale of mine
- ✓ Policy and taxation



2. Valuation Principles

- 1 Based on mineral reserves/resources.
- 2 Considers estimated production profile.
- 3 Includes capital and operating costs.
- 4 Considers metal price, recoveries and market conditions.
- 5 Accounts for time value of money (using discount rate).
- 6 Incorporates risk and uncertainty.
- 7 Common methods: DCF (NPV), Comparative method, Option value (advanced).

3. Cash Flow Components

Inflow (Revenue)	Outflow (Costs)
✓ Sale of minerals (Price × Quantity) ✓ By-product revenue ✓ Royalties (sometimes inflow if refunded) ✓ Incentives / subsidies	✓ Initial capital cost ✓ Operating costs ✓ Mine development cost ✓ Closure and reclamation costs ✓ Taxes and royalties ✓ Working capital ✓ Sustaining capital cost

4. Key Formulas

1 Net Present Value (NPV):

$$NPV = \sum_{t=0}^n \frac{(R_t - C_t)}{(1 + i)^t}$$

R_t = Revenue in year t
 C_t = Cost in year t
 i = discount rate
 n = project life (years)



2 Internal Rate of Return (IRR):

$$0 = \sum_{t=0}^n \frac{(R_t - C_t)}{(1 + IRR)^t}$$

3 Payback Period:

$$\text{Payback Period} = \frac{\text{Initial Investment}}{\text{Annual Net Cash Inflow}}$$

5. Costs in Mine Valuation

1 Capital Costs (Initial Investment)

- ✓ Land acquisition
- ✓ Mine development (shafts, ramps, drifts)
- ✓ Plant & machinery
- ✓ Infrastructure (roads, power, water, etc.)
- ✓ Pre-production costs



2 Operating Costs (Annual)

- ✓ Drilling, blasting, loading, hauling
- ✓ Processing and beneficiation
- ✓ Labour, power, fuel, maintenance
- ✓ Administration and overheads
- ✓ Royalties and taxes

3 Sustaining Capital Costs

- ✓ Replacement of equipment
- ✓ Ongoing development and expansions

4 Closure and Reclamation Costs

- ✓ Environmental management
- ✓ Site rehabilitation and closure liabilities

6. Revenue Estimation

$$\text{Revenue} = Q \times G \times R \times \text{Rec}$$

Q = Quantity of ore/metal produced (tonnes)
 G = Grade of mineral (%)
 R = Metal/commodity price (₹/tonne)
 Rec = Recovery (fraction)



Example:

Annual production = 1 million tonnes
 Grade = 1.5% Cu
 Cu price = ₹ 700,000 per tonne
 Recovery = 90%

$$\text{Revenue} = 1,000,000 \times 0.015 \times 700,000 \times 0.90 = ₹ 9,450 \text{ crore per year}$$

7. Profitability Indicators

Indicator	Formula	Meaning
NPV	$\sum_{t=0}^n \frac{CF_t}{(1 + i)^t}$	Present value of all cash flows
IRR	Value of i for which $NPV = 0$	Rate of return of the project
Payback Period	$\frac{\text{Initial Investment}}{\text{Annual Net Cash Inflow}}$	Time to recover initial investment
Profitability Index (PI)	$PI = \frac{\text{PV of Inflows}}{\text{PV of Outflows}}$	$PI > 1 \rightarrow \text{Accept}$ $PI < 1 \rightarrow \text{Reject}$
Net Annual Cash Flow (CF)	$CF = \text{Revenue} - \text{Operating Costs} - \text{Taxes} \pm \text{Working Capital}$	Annual cash generated by the project

8. Numerical Example - NPV

A mine project has the following cash flows. Discount rate = 10%. Find NPV.

Year (t)	Revenue (₹ crore)	Operating Cost (₹ crore)	Net Cash Flow (₹ crore)
0	-	(-)	-500
1	300	150	150
2	320	160	160
3	350	170	180
4	280	150	130
5	200	140	60

Solution:

$$NPV = \sum_{t=0}^5 \frac{CF_t}{(1 + 0.10)^t}$$

$$= -500 + \frac{150}{1.1} + \frac{160}{1.1^2} + \frac{180}{1.1^3} + \frac{130}{1.1^4} + \frac{60}{1.1^5}$$

$$NPV = ₹ 3.74 \text{ crore (Positive} \rightarrow \text{Accept)}$$

9. Numerical Example - IRR

Find IRR for the following cash flows.

Year (t)	Cash Flow (₹ crore)
0	-400
1	120
2	160
3	180
4	140

Solution (Trial and Error / Calculator):

At $i = 20\%$, $NPV = +15.2$

At $i = 25\%$, $NPV = -8.7$

Interpolating,

$$IRR \approx 20 + \frac{15.2}{15.2 + 8.7} \times 5$$

$$IRR \approx 23.2\%$$



10. Key Takeaways

- ✓ Mine valuation is based on future cash flows.
- ✓ NPV is the most widely used method.
- ✓ Consider all costs (initial, operating, sustaining, closure).
- ✓ Use realistic metal prices, grade and recovery.
- ✓ Profitability indicators help in decision making.
- ✓ Always consider risk, uncertainty and sensitivity analysis.

"A Well-Valued
Mine Leads to
Well-Planned Growth
and Sustainable
Mining."

11. Important Points for GATE

- ✓ Understand NPV, IRR, Payback and PI formulas.
- ✓ Be comfortable with cash flow calculations.
- ✓ Include all relevant costs and revenues.
- ✓ Consider effect of discount rate.
- ✓ Practice numerical problems on NPV and IRR.
- ✓ Interpret results for project decision (accept/reject).
- ✓ Know assumptions and limitations.

Mineral Taxation

"Concepts, Royalties, Taxes, Government Revenue & Effect on Mine Economics"

1. What is Mineral Taxation?

- Mineral taxation refers to the imposition of various levies, taxes and royalties by the government on the extraction and utilization of mineral resources.
- It is a major source of government revenue and helps in resource management and sustainable mining.



2. Objectives

- ✓ Generate revenue for the government
- ✓ Regulate mineral production
- ✓ Ensure optimal and sustainable utilization of resources
- ✓ Discourage over-extraction
- ✓ Support regional development
- ✓ Compensate for depletion of non-renewable resources
- ✓ Promote environmental protection

3. Major Components

- 1 Royalty (based on quantity or value)
- 2 Dead Rent
- 3 District Mineral Foundation (DMF)
- 4 National Mineral Exploration Trust (NMET)
- 5 Goods and Services Tax (GST)
- 6 Income Tax / Corporate Tax
- 7 Cess and Other Levies

4. Royalty

- Royalty is the primary tax on mineral production paid to the government.
- It is either ad valorem (percentage of value) or specific (per unit quantity).
- Rates are specified in the Mines and Minerals (Development and Regulation) Act and rules.

Types:

- 1 Ad valorem royalty = % of value
- 2 Specific royalty = Fixed amount per unit

Royalty Calculation

$$\text{Royalty} = Q \times R$$

Q = Quantity of mineral (tonnes)
R = Royalty rate (₹/tonne or % of value)

Example:

Production of iron ore = 1,00,000 tonnes
Royalty rate = ₹300 per tonne
Royalty = $1,00,000 \times 300$
= ₹ 3,00,00,000 (₹ 3 crore)

5. Dead Rent

- Additional amount payable by leaseholder for minerals extracted beyond the normal rate of extraction.
- Applicable when production exceeds a specified limit.

$$\text{Dead Rent} = (Q_{\text{actual}} - Q_{\text{normal}}) \times R_{\text{dr}}$$

Q_{actual} = Actual production (tonnes)
 Q_{normal} = Normal production (tonnes)
 R_{dr} = Dead rent rate (₹/tonne)

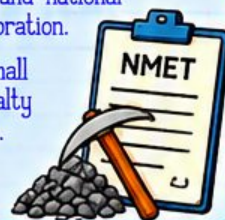
6. DMF (District Mineral Foundation)

- Collected from mining companies for the welfare of people affected by mining activities.
- Used for health, education, land rehabilitation, drinking water, skill development etc.

Typically a % of royalty (usually 10%-30% depending on mineral and state rules).

7. NMET (National Mineral Exploration Trust)

- Collected for mineral exploration and related activities.
- Used for regional and national level mineral exploration.
- Rate: Usually a small percentage of royalty (e.g., 2% of royalty).



8. GST (Goods and Services Tax)

- Applicable on sale of minerals (when supplied in the course of business).
- GST rate varies with mineral type.
- Collected in addition to royalty, DMF and NMET.

Example GST Rates (approx):

- ✓ Coal: 5%
- ✓ Iron ore: 5%
- ✓ Limestones: 5%
- ✓ Bauxite: 5%
- ✓ Granite: 18%
- ✓ Others: 5-18%

9. Income Tax / Corporate Tax

- Profit earned from mining is subject to Income Tax / Corporate Tax (as per prevailing rates).
- Allowable deductions include: operating costs, depreciation, interest, royalty, DMF, NMET etc.



10. Effect on Mine Economics

Component	Effect on Project
Royalty	Increases cost of production
Dead rent	Higher cost for increased production
DMF & NMET	Increases total tax burden
GST	Adds to selling cost
Income tax	Reduces net profit
Higher taxation	May affect project feasibility, NPV, IRR and payback period

11. Numerical Example

A mine produces 5,00,000 tonnes of limestone. Sale price = ₹800 per tonne. Royalty = 10% of value. DMF = 30% of royalty. NMET = 2% of royalty. GST = 5% on sale value. Find total tax and government revenue.

Particulars	Calculation	Amount (₹)
Sale value	$5,00,000 \times 800$	40,00,00,000
Royalty (10%)	$10\% \times 40,00,00,000$	4,00,00,000
DMF (30%)	$30\% \times 4,00,00,000$	1,20,00,000
NMET (2%)	$2\% \times 4,00,00,000$	8,00,000
GST (5%)	$5\% \times 40,00,00,000$	2,00,00,000
Total Govt. Revenue		7,28,00,000

12. Key Takeaways

- ✓ Royalty is the main mineral tax.
- ✓ Dead rent applies for excess production.
- ✓ DMF and NMET support welfare and exploration.
- ✓ GST and Income Tax add to total tax burden.
- ✓ Taxation affects project feasibility and profitability.
- ✓ Always consider all taxes in economic evaluation.

"Mineral taxation balances resource use, government revenue and sustainable development."

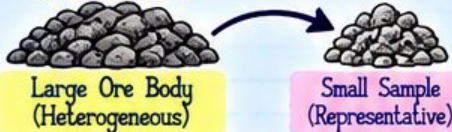


Sampling Methods & Practices

"Sampling Principles, Methods, Sample Preparation, Representativeness, Bias, Precision & Errors"

1. Why Sampling?

- It is not possible to analyse the entire ore/rock mass.
- Sampling is done to obtain a small representative portion to estimate the grade and quality.
- Helps in reserve estimation, mine planning, process design and valuation.



2. Principles of Sampling

- The sample must be representative of the entire lot.
- Every part of the material must have an equal chance of being selected.
- Sample must be collected, handled and prepared without bias.
- Correct sample size and method should be used.
- Sampling must follow systematic and standard procedures.

3. Characteristics of a Good Sample

- ✓ Representative of the population
- ✓ Adequate size (sufficient quantity)
- ✓ Collected without bias
- ✓ Properly preserved and labelled
- ✓ Free from contamination
- ✓ Suitable for intended analysis
- ✓ Economical and practical



4. Sampling Methods

1. Grab Sampling

- Random collection of pieces
- Quick and simple
- Used for preliminary study



2. Channel Sampling

- Continuous sample from a channel
- Used in exposed faces, trenches
- Gives better representation



3. Chip Sampling

- Small chips collected at regular intervals
- Used in underground workings
- Common in exploration



4. Core Sampling

- Cylindrical core obtained by drilling
- Most reliable method
- Used in exploration and detailed



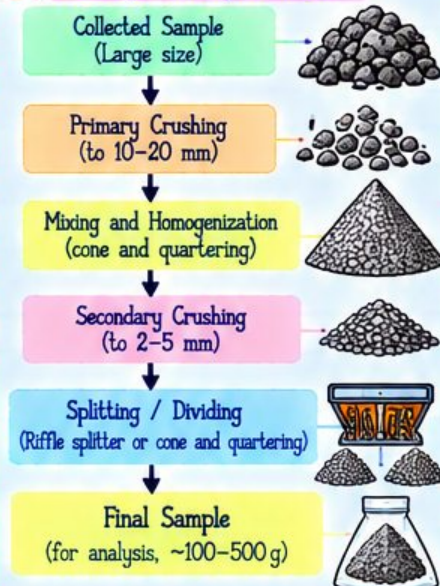
5. Bulk Sampling

- Large quantity sample
- Used for metallurgical testing
- High cost but more reliable

5. Sampling Practice (Field Procedure)

- Define the sampling objective.
- Select appropriate sampling method.
- Collect sample at regular intervals and from fresh surfaces.
- Avoid weathered, altered or contaminated material.
- Ensure proper sample size.
- Label the sample with location, depth, date and other details.
- Store and transport carefully to laboratory.

6. Sample Preparation



7. Representativeness

- A representative sample must have the same composition and grade as the whole population.
- Depends on: sampling method, sample size, heterogeneity, and correct handling.



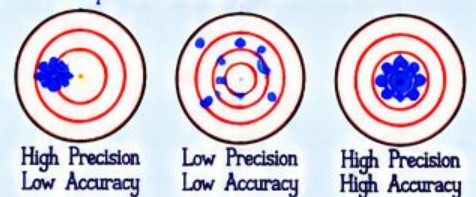
8. Bias in Sampling

- Bias occurs when certain parts of the material are preferentially included or excluded.
- Causes:
 - Incorrect sampling method
 - Human error
 - Segregation during handling
 - Contamination
 - Selective sampling

Result: Sample does not represent the true grade and leads to wrong decisions.

9. Precision and Accuracy

- Precision:** Degree of repeatability of results (low variability).
- Accuracy:** Closeness of the sample result to the true value.



10. Sampling Errors

- Fundamental Error:** Natural variability of the deposit (cannot be eliminated).
- Grouping and Segregation Error:** Due to particle size distribution.
- Sampling Error:** Due to incorrect method or insufficient sample size.
- Preparation Error:** Loss of fines or segregation during crushing and splitting.
- Analytical Error:** Due to laboratory and instrumental limitations.

11. Factors Affecting Sampling

- ✓ Heterogeneity of the deposit
 - ✓ Particle size variation
 - ✓ Sampling method used
 - ✓ Sample size and frequency
 - ✓ Proper sample preparation
 - ✓ Skill and experience of personnel
 - ✓ Environmental conditions (weathering, moisture, contamination)
- Rule of Thumb:** Larger and more heterogeneous the material, larger the sample size required.

12. Key Takeaways

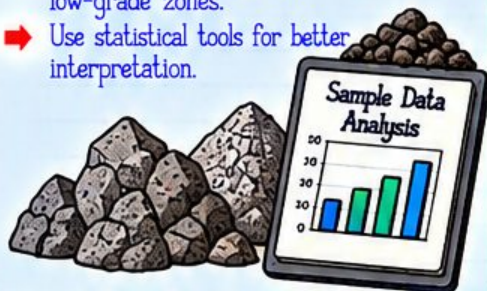
- ✓ Sampling is essential for estimating grade and quality.
- ✓ Use appropriate sampling method based on purpose.
- ✓ Ensure representativeness, avoid bias and maintain precision.
- ✓ Follow proper sample preparation steps.
- ✓ Understand sources of sampling errors and minimize them.
- ✓ Accurate sampling leads to reliable resource estimation and better mine planning.

Sampling Interpretation & Quality Control

"Sample Data Interpretation, Grade Variability, QA/QC, Blanks, Standards, Duplicates, Assay Quality & Validation"

1. Sample Data Interpretation

- ➔ Analyse sample data to estimate grade, quality and variability.
- ➔ Identify trends, high-grade and low-grade zones.
- ➔ Use statistical tools for better interpretation.



2. Grade Variability

- ➔ Grade varies due to natural heterogeneity of the ore deposit.
- ➔ Affects the number of samples required and confidence in results.
- ➔ Measured using statistical tools like mean, variance and coefficient of variation (CV).



Coefficient of Variation (CV)

$$CV = \frac{S}{\bar{X}} \times 100\%$$

S = Standard deviation

$\bar{X}$ = Mean grade

3. Statistical Analysis

- ✓ Mean (Average grade)

$$\bar{X} = \frac{\sum X_i}{n}$$

- ✓ Variance

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n - 1}$$

- ✓ Standard Deviation

$$S = \sqrt{S^2}$$

- ✓ Coefficient of Variation

$$CV = \frac{S}{\bar{X}} \times 100\%$$

- ✓ Range

$$R = X_{\max} - X_{\min}$$

4. Data Interpretation Example

Assay results of 8 samples from a borehole are given below. Calculate mean grade, standard deviation and coefficient of variation.

Sample No.	Grade (% Cu)	Sample No.	Grade (% Cu)
1	0.8	5	1.8
2	1.2	6	1.1
3	1.0	7	1.3
4	1.5	8	1.0

Solution:

$$\bar{X} = 1.275 \% \text{ Cu}$$

$$S = 0.324$$

$$CV = 25.4\%$$

Interpretation:

CV = 25.4% indicates moderate grade variability.

5. Quality Assurance (QA) vs Quality Control (QC)

Quality Assurance (QA)

- ✓ Systematic planning and procedures to ensure reliable results.
- ✓ Involves sampling design, sample preparation, documentation, standards and audits.
- ✓ Prevents errors.
- ✓ Focus on overall process quality.

Quality Control (QC)

- ✓ Checks the accuracy and precision of the assay results.
- ✓ Includes use of blanks, standards, duplicates and replicate samples.
- ✓ Identifies errors and ensures data reliability.
- ✓ Focus on laboratory and analytical quality.

6. QA/QC Tools

- 1 **Blanks** - Check for contamination.
- 2 **Standards** - Check for accuracy.
- 3 **Duplicates** - Check for precision.
- 4 **Replicates** - Repeat analysis of the same sample.
- 5 **Pulp Duplicates** - Check laboratory precision.
- 6 **Field Duplicates** - Check sampling precision.



7. Blanks

- ➔ Material with no value of the element (e.g., barren rock, quartz).
- ➔ Inserted regularly in the sample sequence.
- ➔ Used to detect contamination during sample preparation and analysis.

Blank result should be close to zero.
High value indicates contamination.

8. Standards (Reference Materials)

- ➔ Material with known grade/assay value.
- ➔ Used to check accuracy of assay results.
- ➔ Inserted at regular intervals (e.g., 1 in 20 samples).
- ➔ Assay result should be within acceptable range (e.g., ± 2 standard deviations).

If result is outside the control limits, then investigation and corrective action is required.

9. Duplicates

- ➔ Two samples from the same material.
- ➔ Used to check precision (repeatability).
- ➔ Can be field duplicate, coarse reject duplicate, pulp duplicate or assay duplicate.

Relative Percent Difference (RPD)

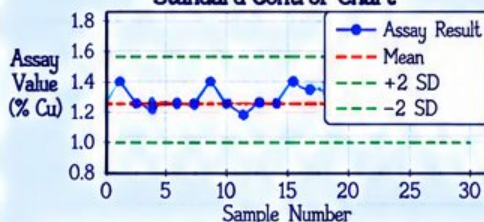
$$RPD = \frac{|X_1 - X_2|}{(X_1 + X_2)/2} \times 100\%$$

Acceptable RPD typically < 10–20% (depending on grade level).

10. QC Control Charts

Used to monitor assay results of standards and blanks over time.

Standard Control Chart



Most results should lie within ± 2 SD limits.

11. Assay Quality and Validation

- ✓ Check for outliers and inconsistent results.
- ✓ Compare duplicates, standards and blanks.
- ✓ Use statistical analysis (mean, SD, CV, RPD).
- ✓ Validate data before using for resource estimation, mine planning and valuation.
- ✓ Remove or re-assay incorrect samples.

Common Problems

- ✗ High blank values (contamination)
- ✗ Standard out of range (accuracy issue)
- ✗ High RPD (poor precision)
- ✗ Missing samples / incorrect labelling
- ✗ Data transcription errors

12. Key Takeaways

- ✓ Interpret sample data using statistical tools.
- ✓ Grade variability affects sampling density and confidence.
- ✓ Use QA/QC tools: blanks, standards, duplicates.
- ✓ Maintain proper sample preparation and handling.
- ✓ Validate assay results before use.
- ✓ Reliable data leads to better resource estimation and mine planning decisions.

Reserve Estimation Techniques

"Polygonal Method, Triangular Method, Sectional Methods, Block Methods, Average Grade & Tonnage Calculations"

1. What is Reserve Estimation?

- Estimation of quantity (tonnage) and quality (grade) of mineral present in a defined area/volume.
- Used for mine planning, production scheduling and economic evaluation.

Produced as: Tonnage (T)
Grade (G)
Metal content (T × G)



2. Basic Formulas

① Tonnage (T):
 $T = \text{Volume (V)} \times \text{Density } (\rho)$
 $V = \text{Area} \times \text{Thickness}$

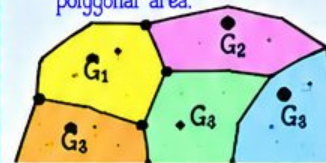
T = Tonnage (tonnes)
V = Volume (m³)
ρ = Density (t/m³)
G = Grade (% or g/t)
M = Metal content
T_i = Tonnage of ith block
G_i = Grade of ith block

② Metal Content (M):
 $M = T \times G$

③ Average Grade (G_{avg}):
 $G_{avg} = \frac{\sum (T_i \times G_i)}{\sum T_i}$

3. Polygonal Method

- Area is divided into polygons around sample points.
- Each sample is assumed to represent the grade of its polygonal area.

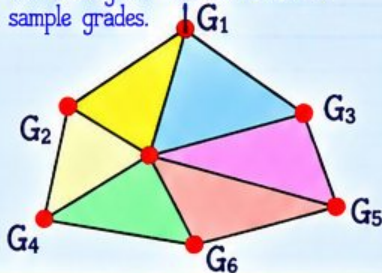


$$T_i = A_i \times t \times \rho$$

$$G_{avg} = \frac{\sum (A_i \times t \times \rho \times G_i)}{\sum (A_i \times t \times \rho)}$$

4. Triangular Method

- Area is divided into triangles by joining adjacent sample points.
- Each triangle grade is taken as the average of the three corner sample grades.

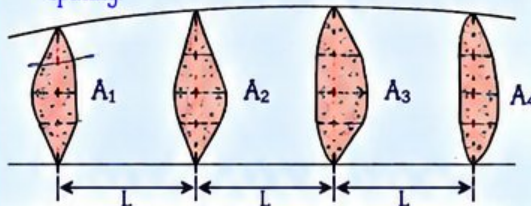


$$T_i = \left(\frac{1}{2} A_{\triangle}\right) \times t \times \rho$$

$$G_{\triangle} = \frac{G_1 + G_2 + G_3}{3}$$

5. Sectional Method (Cross-Sectional / End Area)

- Used when the deposit has irregular shape along strike and dip.
- Cross-sections are drawn at regular intervals. Area of ore is calculated for each section and volume is obtained by multiplying with section spacing.

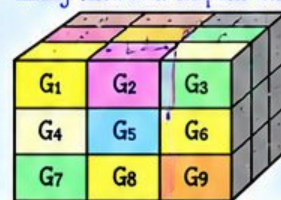


$$T = \frac{A_1 + A_2 + A_3 + A_4}{2} \times L \times \rho$$

$$G_{avg} = \frac{A_1 G_1 + A_2 G_2 + A_3 G_3 + A_4 G_4}{A_1 + A_2 + A_3 + A_4}$$

6. Block Method

- Deposit is divided into 3D blocks (cells) of equal size.
- Each block is assigned a grade (assay result or estimated).
- Most common method in modern mining (used with computer software).



$$T_i = V_i \times \rho$$

$$G_{avg} = \frac{\sum (T_i \times G_i)}{\sum T_i}$$

7. Example - Polygonal Method

Calculate tonnage and average grade.

Polygon	Area (m²)	Thickness (m)	Density (t/m³)	Grade (% Cu)
1	500	10	2.6	1.2
2	300	10	2.6	0.8
3	700	10	2.6	1.5

Solution: $T_1 = 500 \times 10 \times 2.6 = 13,000 \text{ t}$

$T_2 = 300 \times 10 \times 2.6 = 7,800 \text{ t}$

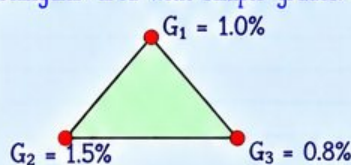
$T_3 = 700 \times 10 \times 2.6 = 18,200 \text{ t}$

Total T = 39,000 t

$$G_{avg} = \frac{(13,000 \times 1.2 + 7,800 \times 0.8 + 18,200 \times 1.5)}{39,000} = 1.26 \% \text{ Cu}$$

8. Example - Triangular Method

Triangular area with sample grades.



Area of triangle = 1,000 m²

Thickness = 10 m

Density = 2.6 t/m³

$$\text{Grade} = \frac{(1.0 + 1.5 + 0.8)}{3} = 1.1 \%$$

Tonnage = 1,000 × 10 × 2.6 = 26,000 t

Metal content = 26,000 × 1.1% = 286 t

9. Example - Sectional Method

Three cross-sections at 50 m interval.

Section	Area (m²)	Grade (% Cu)
1	800	1.0
2	1,200	1.5
3	1,000	1.2

Spacing (L) = 50 m, Density = 2.6 t/m³

$$\text{Tonnage} = \frac{(800 + 1,200 + 1,000)}{2} \times 50 \times 2.6 = \frac{3,000}{2} \times 50 \times 2.6 = 195,000 \text{ t}$$

$$\text{Average grade} = \frac{(800 \times 1.0 + 1,200 \times 1.5 + 1,000 \times 1.2)}{3,000} = \frac{800 + 1,800 + 1,200}{3,000} = 1.27 \% \text{ Cu}$$

10. Example - Block Method

Block model (equal block size 10×10×10 m), ρ = 2.6 t/m³.

Block	Grade (% Cu)	Volume of each block = 10 × 10 × 10 = 1,000 m³
1	1.0	Tonnage per block = 1,000 × 2.6 = 2,600 t
2	1.5	Total T = 4 × 2,600 = 10,400 t
3	0.8	Average grade = $\frac{1.0 + 1.5 + 0.8 + 1.2}{4} = 1.125 \%$
4	1.2	Metal content = 10,400 × 1.125% = 117 t

11. Key Takeaways

- ✓ Choose method based on deposit geometry, data availability and accuracy required.
- ✓ Polygonal and triangular methods - simple and widely used.
- ✓ Sectional method - suitable for elongated and irregular deposits.
- ✓ Block method - most accurate, widely used in modern mining.
- ✓ Always consider density, grade variability and data quality.
- ✓ Calculate both tonnage and average grade.

12. Common Sources of Error

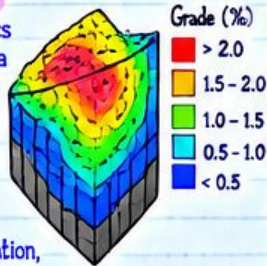
- ✗ Inaccurate sample grades
- ✗ Incorrect area/volume measurement
- ✗ Improper section spacing
- ✗ Assuming uniform grade within large area/block
- ✗ Ignoring grade variability
- ✗ Human and instrumental errors
- ✗ Poor geological interpretation

Geostatistics - Fundamentals

"Variability, Regionalized Variables, Spatial Correlation, Variogram, Nugget, Sill, Range, Basic Geostatistical Concepts"

1. What is Geostatistics?

- Geostatistics is the branch of statistics that deals with spatially correlated data and its application in natural resources evaluation and estimation.
- It helps to model and predict the spatial distribution of variables (e.g., grade, metal content) in the earth's crust.
- Widely used in mineral resource estimation, mining, petroleum, and environmental studies.



Key Idea : Nearby samples are more similar than distant samples.

2. Variability

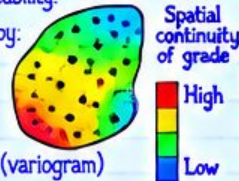
- Natural deposits show spatial and statistical variability due to geological processes.
- Measured using:
 - ✓ Mean (average)
 - ✓ Variance (spread)
 - ✓ Standard deviation
 - ✓ Coefficient of variation (CV)

$$CV = \frac{S}{\bar{X}} \times 100\%$$

S = Standard deviation
X̄ = Mean grade

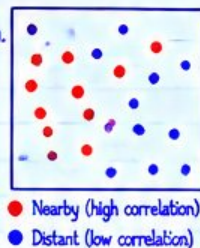
3. Regionalized Variables

- A regionalized variable is a variable that shows spatial continuity and correlation over a region.
- Examples: Ore grade, thickness, porosity, permeability.
- Characterized by:
 - ✓ Mean
 - ✓ Variance
 - ✓ Spatial correlation (variogram)



4. Spatial Correlation

- Measures how values at one location are related to values at another location.
- Expressed using the variogram or covariance function.
- Nearby samples → high correlation (positive).
- Distant samples → low correlation (approaches zero).



More continuity = Stronger correlation
More distance = Weaker correlation

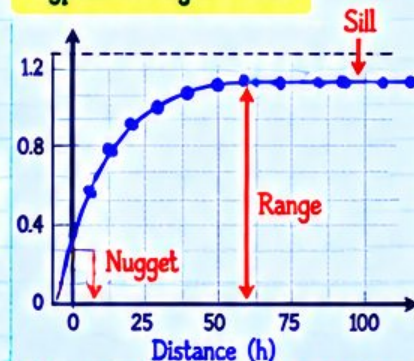
5. Variogram

- A variogram shows how data variability increases with distance.
- It is the most common function used in geostatistics.

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [Z(x_i) - Z(x_i + h)]^2$$

$\gamma(h)$ = semivariance
• h = lag distance
N(h) = number of pairs

Typical Variogram Model



6. Nugget, Sill and Range

Nugget

- Value at zero distance.
- Represents measurement error, small-scale variability or unidentified factors.
- Unit: same as the variable (e.g., % grade).

Sill

- Plateau value of the variogram.
- Total variance = structured + nugget.
- Unit: same as the variable.

Range

- Distance where variogram reaches sill.
- Beyond this distance, samples are not correlated.
- Unit: same as distance (e.g., meters).

7. Basic Geostatistical Concepts

1 Kriging

- Best linear unbiased estimator (BLUE).
- Uses variogram and spatial correlation.

2 Block Kriging

- Used for grade and tonnage estimation in blocks.
- Accounts for block size and geometry.

3 Indicator Kriging

- Used for categorical data (e.g., presence/absence).
- Based on probability.

4 Co-kriging

- Uses two or more correlated variables.
- Improves estimation accuracy.

8. Key Takeaways

- ✓ Geostatistics models spatial continuity and variability.
- ✓ Variogram is the key tool.
- ✓ Nugget = short scale variability / measurement error.
- ✓ Sill = total variance.
- ✓ Range = correlation distance.
- ✓ Kriging gives the best estimate using spatial correlation.
- ✓ Essential for accurate reserve estimation and mine planning.

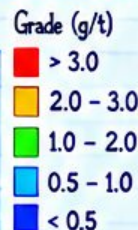
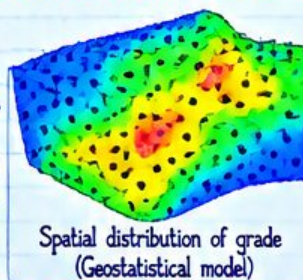
"Better Data

- Better Models
- Better Decisions"



Real World Example

In a gold deposit, grade varies spatially. Using variogram analysis, we can estimate the grade at unsampled locations more accurately by considering the spatial correlation between nearby samples.



Remember:

- Variability is natural.
- Correlation exists.
- Variogram shows it.
- Kriging uses it. 😊

Geostatistics
+
Good Data
=
Reliable
Reserve Estimate

Reserve Estimation + Geostatistics Numericals

"Grade Estimation, Weighted Averages, Variogram Interpretation, Basic Estimation Problems & GATE-level Numericals"

1. Types of Estimation

- ① Simple average method
- ② Weighted average method
- ③ Polygonal / Triangular / Sectional methods
- ④ Block method
- ⑤ Geostatistical estimation (inverse distance, kriging)

2. Weighted Average Grade

$$G_{avg} = \frac{\sum (T_i \times G_i)}{\sum T_i}$$

Where,

T_i = tonnage of block/section

G_i = grade of block/section

3. Inverse Distance Weighting (IDW)

$$Z(x_0) = \frac{\sum (w_i \times Z(x_i))}{\sum w_i}$$

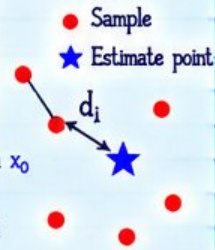
$$w_i = \frac{1}{d_i^p}$$

$Z(x_0)$ = estimated grade at location x_0

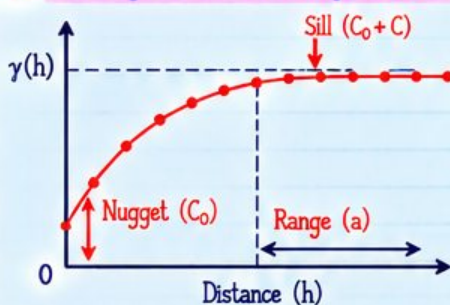
$Z(x_i)$ = grade at sample point i

d_i = distance between x_0 and x_i

p = power (commonly 1 or 2)



4. Variogram Model Interpretation



- Nugget (C_0) : Variance at zero distance (measurement error + micro-variability)
- Sill ($C_0 + C$) : Total variance (plateau value)
- Range (a) : Distance beyond which samples are not correlated
- Model types : Spherical, Exponential, Gaussian

5. Example 1 - Weighted Average Grade

A deposit is divided into 4 blocks with the following tonnages and grades. Find the overall grade.

Block	Tonnage (t)	Grade (%Cu)
1	10,000	1.2
2	20,000	0.8
3	15,000	1.5
4	5,000	1.0

Solution:

$$\sum (T_i G_i) = 10,000 \times 1.2 + 20,000 \times 0.8 + 15,000 \times 1.5 + 5,000 \times 1.0 = 12,000 + 16,000 + 22,500 + 5,000 = 55,500$$

$$\sum T_i = 10,000 + 20,000 + 15,000 + 5,000 = 50,000$$

$$G_{avg} = \frac{55,500}{50,000} = 1.13 \% \text{ Cu}$$

6. Example 2 - IDW Estimation

Estimate the grade at point P using IDW ($p = 2$) from 3 nearby samples.

Sample	Distance (m)	Grade (%Cu)
A	100	1.0
B	200	1.5
C	300	0.8

Solution:

$$w_i = \frac{1}{d_i^2}$$

$$w_A = \frac{1}{100^2} = 0.0001$$

$$w_B = 1/200^2 = 0.000025$$

$$w_C = 1/300^2 = 0.0000111$$

$$Z(P) = \frac{\sum w_i Z_i}{\sum w_i}$$

$$Z(P) = \frac{(0.0001 \times 1.0 + 0.000025 \times 1.5 + 0.0000111 \times 0.8)}{(0.0001 + 0.000025 + 0.0000111)}$$

$$= \frac{0.000174}{0.0001361} = 1.28 \% \text{ Cu}$$

7. Example 3 - Block Grade Estimation

A block is influenced by 4 surrounding samples using IDW ($p = 2$). Find estimated grade.

Sample	Distance (m)	Grade (%Cu)
1	150	0.9
2	200	1.4
3	250	1.1
4	300	0.8

Solution:

$$w_1 = 1/150^2 = 0.0000444$$

$$w_2 = 1/200^2 = 0.0000250$$

$$w_3 = 1/250^2 = 0.0000160$$

$$w_4 = 1/300^2 = 0.0000111$$

$$Z = \frac{\sum w_i Z_i}{\sum w_i}$$

$$Z = \frac{(0.0000444 \times 0.9 + 0.000025 \times 1.4 + 0.000016 \times 1.1 + 0.0000111 \times 0.8)}{(0.0000444 + 0.000025 + 0.000016 + 0.0000111)}$$

$$= \frac{0.000107}{0.0000965} = 1.11 \% \text{ Cu}$$

8. Example 4 - Variogram Interpretation

A spherical variogram model has $C_0 = 0.2$, $C = 1.0$ and range (a) = 300 m. Find $\gamma(150 \text{ m})$ and $\gamma(400 \text{ m})$.

Spherical Model:

$$\gamma(h) = \begin{cases} C_0 + C \left[1.5 \frac{h}{a} - 0.5 \left(\frac{h}{a} \right)^3 \right], & 0 < h \leq a \\ C_0 + C, & h > a \end{cases}$$

Solution:

➤ For $h = 150 \text{ m}$ ($h < a$)

$$\begin{aligned} \gamma(150) &= 0.2 + 1.0 \left[1.5 \left(\frac{150}{300} \right) - 0.5 \left(\frac{150}{300} \right)^3 \right] \\ &= 0.2 + 1.0 [0.75 - 0.5 (0.125)] \\ &= 0.2 + 1.0 [0.75 - 0.0625] \\ &= 0.2 + 0.6875 = 0.8875 \end{aligned}$$

➤ For $h = 400 \text{ m}$ ($h > a$)

$$\gamma(400) = C_0 + C = 0.2 + 1.0 = 1.2$$

9. Example 5 - Sectional Tonnage & Grade

A deposit has 3 cross-sections 100 m apart. Find total tonnage and average grade.

Section	Area (m^2)	Grade (%Cu)
1	800	1.0
2	1,200	1.5
3	1,000	1.2

Solution:

➤ Volume (Trapezoidal rule)

$$\begin{aligned} V &= \frac{h}{2} [A_1 + 2A_2 + A_3] \\ &= \frac{100}{2} [800 + 2(1,200) + 1,000] \\ &= 50 [800 + 2,400 + 1,000] \\ &= 50 \times 4,200 = 210,000 \text{ m}^3 \end{aligned}$$

➤ Tonnage = $V \times \rho$ ($\rho = 2.6 \text{ t/m}^3$)

$$= 210,000 \times 2.6 = 546,000 \text{ t}$$

➤ Average grade:

$$\begin{aligned} G_{avg} &= \frac{A_1 G_1 + 2A_2 G_2 + A_3 G_3}{A_1 + 2A_2 + A_3} \\ &= \frac{(800 \times 1.0 + 2 \times 1,200 \times 1.5 + 1,000 \times 1.2)}{(800 + 2,400 + 1,000)} \\ &= \frac{4,200}{4,200} = 1.0 \end{aligned}$$

10. GATE-Level Practice Questions

- ① A variogram reaches a sill of 1.1 at 300 m. What is the value of $\gamma(500 \text{ m})$? (A) 0.2 (B) 0.6 (C) 1.1 (D) 1.5
- ② In IDW estimation, increasing the power parameter (p):
(A) gives more weight to distant samples
(B) gives more weight to nearer samples
(C) has no effect
(D) reduces estimation accuracy always
- ③ A deposit is divided into 3 blocks: (T, G) = (10,000, 1.0), (15,000, 1.5), (5,000, 0.8). Find average grade. (NAT)

11. Key Takeaways

- ✓ Choose appropriate estimation method based on data spacing.
- ✓ Use weighted averages for quick estimates.
- ✓ Geostatistics considers spatial correlation.
- ✓ Variogram parameters (nugget, sill, range) help in understanding continuity.
- ✓ IDW is simple and widely used.
- ✓ Kriging gives best linear unbiased estimate (BLUE).
- ✓ Always validate estimates with QA/QC and geological understanding.
- ✓ Practice numericals for GATE-level accuracy.

12. Common Mistakes

- ✗ Using simple average instead of weighted average.
- ✗ Ignoring spatial correlation.
- ✗ Wrong variogram parameter interpretation.
- ✗ Using inappropriate power (p) in IDW.
- ✗ Calculation errors in tonnage and grade.
- ✗ Not checking units (m, m^3 , t, %).

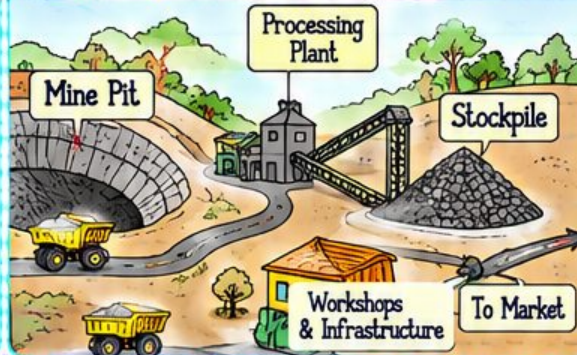
Optimization of Facility Location

"Facility-Location Principles, Haulage Distance, Transportation Cost, Processing Facilities, Stockpiles, Infrastructure-Location Factors"

1. What is Facility Location?

- Deciding the best location for mine facilities (processing plant, stockpiles, workshops, etc.)
- Aim: minimum total cost, maximum efficiency and smooth operations.
- Involves trade-off between transportation cost, construction cost, operating cost and environmental / social factors.

2. Key Facilities in a Mine



3. Objectives of Facility Location

- Minimise total cost (transportation + construction + operating).
- Minimise haulage distance.
- Ensure adequate capacity and space for future expansion.
- Improve operational efficiency.
- Consider environmental and social impacts.
- Ensure availability of water, power, roads and manpower.

4. Haulage Distance and Transportation Cost



$$\text{Transportation Cost } (C_t) = T \times D \times C$$

T = Annual tonnage (t)

D = Haulage distance (km)

C = Cost of transport per tonne per km (₹/t.km)

5. Optimum Location (Weighted Center Method)

$$X^* = \frac{\sum W_i x_i}{\sum W_i} \quad Y^* = \frac{\sum W_i y_i}{\sum W_i}$$

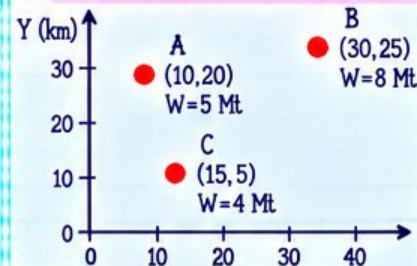
W_i = tonnage from source i (or annual production/flow)

(x_i, y_i) = coordinates of source i

Steps:

- Identify sources (pits/deposits).
- Use tonnage as weight (W_i).
- Calculate X^* and Y^* .
- Locate plant near (X^*, Y^*) .

6. Example - Optimum Plant Location



Solution:

$$X^* = \frac{5 \times 10 + 8 \times 30 + 4 \times 15}{5 + 8 + 4} = \frac{370}{17} = 21.76 \text{ km}$$

$$Y^* = \frac{5 \times 20 + 8 \times 25 + 4 \times 5}{17} = \frac{415}{17} = 24.41 \text{ km}$$

Optimum plant location $\approx (21.8, 24.4)$ km

7. Factors Affecting Facility Location

- Proximity to mine (haulage distance)
- Availability of water supply
- Power supply (electricity / fuel)
- Accessibility (road / rail / port)
- Topography and site conditions
- Land availability and cost
- Environmental and social constraints
- Scope for future expansion
- Labour availability
- Government regulations and permits

8. Processing Facility Location

- Should be close to mine to reduce transportation cost.
- Adequate land for plant, tailings disposal and future expansion.
- Consider water and power availability.
- Favourable topography and geology.
- Minimise environmental impact (dust, noise, effluent).
- Consider by-product recovery and market access.

$$\text{Total Cost} = C_{\text{mining}} + C_{\text{haulage}} + C_{\text{processing}} + C_{\text{infrastructure}} + C_{\text{environmental}}$$

9. Stockpile Location

- Act as a buffer between mining and plant.
- Should be near the plant to minimise re-handling cost.
- Adequate area for required capacity.
- Proper drainage and environmental control.
- Useful for grade blending and continuous plant operation.



10. Comparative Analysis of Alternative Sites

Factor	Site A	Site B	Site C
Distance from mine (km)	15	25	20
Land cost (₹ million)	50	30	40
Transportation cost (₹/t)	80	120	100
Water availability	Good	Moderate	Good
Power availability	Good	Good	Moderate
Environmental impact	Low	High	Moderate
Total annual cost (₹ million)	250	320	280

Best location: Site A (minimum total cost)

11. Real World Considerations

- Multi-objective optimisation (cost + environment + social).
- Use GIS and computer modelling for large projects.
- Consider rail / port access for export or domestic market.
- Integrate with mine planning and scheduling.
- Allow for future expansion and technological changes.
- Consider stakeholder concerns (local community, government).

12. Key Takeaways

- Facility location aims to minimise total cost and maximise efficiency.
- Transportation cost is a major factor.
- Weighted center method helps in finding optimum location.
- Processing plants and stockpiles should be close to mine and infrastructure.
- Environmental, social and future expansion factors are equally important.
- Use comparative analysis and modern tools (GIS, optimisation) for better decisions.

Mine Planning & Its Components

"Strategic / Tactical / Operational Planning, Mine Layout, Production Scheduling, Infrastructure, Environmental and Economic Considerations"

1. What is Mine Planning?

- ➔ Mine planning is the process of designing and scheduling the extraction of mineral resources in a safe, economic, efficient and environmentally responsible manner.
- ➔ It converts mineral resources into reserves and guides the development and operation of a mine throughout its life.

Objective: Optimum extraction of maximum value with minimum cost, risk and environmental impact.

2. Levels of Mine Planning



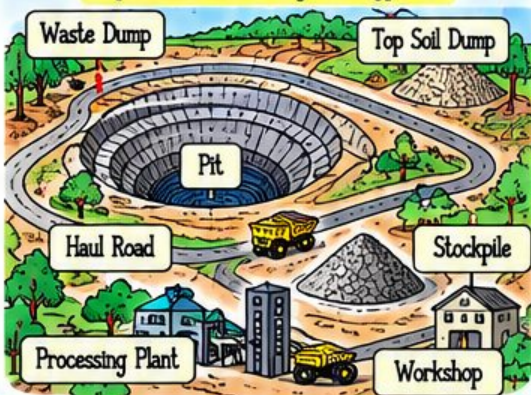
3. Key Components of Mine Planning

- 1 Geological & resource evaluation
- 2 Mine design and layout
- 3 Production scheduling
- 4 Equipment selection and deployment
- 5 Infrastructure planning
- 6 Manpower planning
- 7 Environmental management
- 8 Economic and financial analysis
- 9 Health, safety and risk management
- 10 Closure and reclamation planning

4. Mine Layout

- ➔ Arrangements of pits/underground works, roads, dumps, stockpiles, processing plant, and infrastructure.
- ➔ Should ensure smooth movement of men, material and machinery.
- ➔ A good layout reduces haulage distance, operating cost and environmental impact.

Open-cast Mine Layout (Typical)



5. Production Scheduling

- ➔ Determines when, where and how much material (ore and waste) to mine.
- ➔ Ensures continuous supply of ore to processing plant.
- ➔ Balances production, equipment capacity, manpower and market demand.

Typical Production Schedule

Year	Ore Production (Mt)	Waste Production (Mt)	Strip Ratio	Grade (% Cu)
1	5.0	15.0	3.0	1.2
2	5.5	16.5	3.0	1.1
3	6.0	18.0	3.0	1.0
4	6.0	18.0	3.0	0.9
5	5.5	16.5	3.0	0.8

Key Points:

- ✓ Consider equipment capacity and availability.
- ✓ Maintain consistent ore grade.
- ✓ Optimize strip ratio and cash flow.
- ✓ Adjust for market demand and price fluctuations.

6. Infrastructure Planning

- ➔ Roads (haul and access roads)
- ➔ Power supply (electricity, DG sets)
- ➔ Water supply and dewatering systems
- ➔ Processing plant and beneficiation facilities
- ➔ Workshops, stores and maintenance facilities
- ➔ Housing, offices and amenities for workers
- ➔ Communication and IT systems
- ➔ Rail/port connectivity (for export)

Typical Infrastructure in a Mine



7. Environmental Considerations

- ➔ Minimise land degradation and deforestation.
- ➔ Proper waste dump and overburden management.
- ➔ Control of dust, noise and vibration.
- ➔ Water management and prevention of mine water pollution.
- ➔ Top soil conservation and reclamation.
- ➔ Biodiversity protection.
- ➔ Compliance with environmental regulations (EIA, EMP).

Sustainable Mining



Mine today with responsibility for a better tomorrow.

8. Economic Considerations

- ➔ Evaluate project feasibility (cost-benefit).
- ➔ Capital and operating cost estimation.
- ➔ Revenue estimation based on production and mineral prices.
- ➔ Cash flow analysis and break-even. (Refer to DCF, NPV, IRR discussed earlier)
- ➔ Consider mining method, equipment cost, haulage, processing, infrastructure and environmental cost.

Cost Components

Cost Type	Examples
Capital Cost	Mine development, plant, infrastructure
Operating Cost	Drilling, blasting, loading, hauling, processing
Transportation Cost	Haulage, rail, port charges
Environmental Cost	Mitigation, reclamation, monitoring
Administrative Cost	Manpower, overheads
Closure Cost	Mine closure and rehabilitation

9. Integrated Mine Planning

- ➔ All components should be integrated to ensure technical, economic, environmental and social sustainability.
- ➔ Use of modern tools: GIS, mine planning software (e.g., MineSight, Surpac, Datamine), optimization models.



Mine Size & Mine Life

"Production Capacity, Annual Production, Reserves, Stripping Ratio, Mine Life Calculations, Capacity-Life Relationships"

1. What is Mine Size?

- ➔ Mine size refers to the annual production capacity of a mine.
- ➔ Depends on reserves, market demand, equipment capacity, infrastructure and economics.
- ➔ Can be expressed in million tonnes per annum (Mtpa).

Key Point: Mine size should match market demand and available reserves to ensure continuous and economic operation.

2. What is Mine Life?

- ➔ Mine life is the total period for which a mine can produce at a planned production rate using its reserves.
- ➔ Depends on total reserves, annual production rate and stripping ratio.
- ➔ Includes both ore production and waste removal period.

Key Factors:

- ✓ Total reserves
- ✓ Annual production capacity
- ✓ Stripping ratio
- ✓ Production schedule
- ✓ Market and economic conditions

3. Important Terminologies

- Reserves** : Economically extractable mineral after considering technical and economic factors.
- Resources** : Total estimated mineral occurrence (measured + indicated + inferred).
- Production capacity** : Annual ore production rate.
- Annual production** : Actual ore produced in a year.
- Stripping ratio (SR)** : Ratio of waste tonnage to ore tonnage (W : O).
- Mine life** : Total years of operation at planned production rate.

4. Relationship between Reserves, Production and Mine Life

$$\text{Mine Life (years)} = \frac{\text{Total Recoverable Reserves (tonnes)}}{\text{Annual Production (tonnes/year)}}$$

Example :

Total reserve = 120 million tonnes
Annual production = 6 million tonnes/year

$$\text{Mine life} = \frac{120}{6} = 20 \text{ years}$$

5. Stripping Ratio (SR)

$$\text{Stripping ratio} = \frac{\text{Tonnage of Waste (W)}}{\text{Tonnage of Ore (O)}}$$

- ➔ Expressed as W : O or a numerical value.
- ➔ Higher SR → more waste to be removed → higher cost → shorter mine life.
- ➔ Lower SR → more economic.

Example :

Waste = 240 million tonnes
Ore = 40 million tonnes
SR = 240 / 40 = 6 : 1

6. Effect of Stripping Ratio on Mine Life

- ➔ For a fixed total material (Ore + Waste), higher SR reduces the available ore.
- ➔ More waste handling increases cost and may reduce economic mine life.

Illustration:

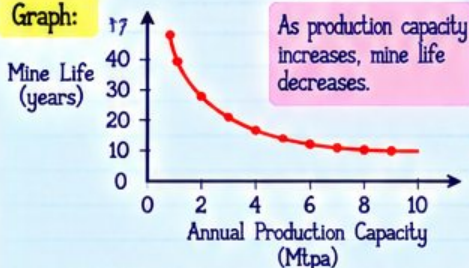
Total material = 100 million tonnes

Stripping Ratio (W:O)	Ore (Mt)	Waste (Mt)	If annual production = 2 Mtpa Mine life (years)
2 : 1	33.3	66.7	16.7
4 : 1	20.0	80.0	10.0
6 : 1	14.3	85.7	7.1

7. Capacity - Life Relationship

- ➔ Higher annual production → shorter mine life.
- ➔ Lower annual production → longer mine life.
- ➔ Choose optimum capacity considering:
 - ✓ Market demand
 - ✓ Capital and operating cost
 - ✓ Equipment availability
 - ✓ Infrastructure and logistics
 - ✓ Discounted cash flow / project economics

Graph:



8. Numerical Problems (GATE Level)

(a) Mine Life Calculation

A mine has 150 million tonnes of economically recoverable reserves. Planned annual production is 5 million tonnes. Find the mine life.

Solution:

$$\text{Mine life} = \frac{150}{5} = 30 \text{ years}$$

(b) Reserves Estimation from Mine Life

A mine is planned for 20 years with an annual production of 3 million tonnes. Find the required reserves.

Solution:

$$\text{Reserves} = 20 \times 3 = 60 \text{ million tonnes}$$

(c) Effect of Stripping Ratio

Total material = 100 million tonnes, SR = 4 : 1. Find ore reserves and mine life if annual production = 2 million tonnes.

Solution:

$$\text{Ore} = \frac{100}{(4+1)} = 20 \text{ million tonnes}$$

$$\text{Mine life} = \frac{20}{2} = 10 \text{ years}$$

10. Real World Example

A coal mine has 300 million tonnes of reserves. Planned production capacity = 10 million tonnes/year. Stripping ratio = 2 : 1. Find mine life and total material to be handled.

Solution:

Mine life = 300 / 10 = 30 years
Waste = 300 × 2 = 600 million tonnes
Total material = 300 + 600 = 900 million tonnes



11. Key Takeaways

- ✓ Mine size and mine life are interdependent.
- ✓ Higher production shortens mine life.
- ✓ Stripping ratio significantly affects economic viability.
- ✓ Optimal capacity should balance market demand, reserves and cost.
- ✓ Use mine planning and economic analysis to decide mine size and life.
- ✓ GATE questions often include calculations on reserves, production, SR and mine life.

12. Common Mistakes

- ✗ Confusing resources with reserves.
- ✗ Ignoring stripping ratio in calculations.
- ✗ Using total resources instead of economically recoverable reserves.
- ✗ Not considering market demand.
- ✗ Wrong units (Mt, t, m³).
- ✗ Assuming constant grade and recovery.
- ✗ Ignoring equipment and infrastructure limitations.
- ✗ Not checking economic feasibility.

Ultimate Pit Design

"Open-pit Geometry, Pit Limits, Slope Considerations, Bench Parameters, Stripping Ratio, Ultimate Pit Concept and Design Principles"

1. What is Ultimate Pit Design?

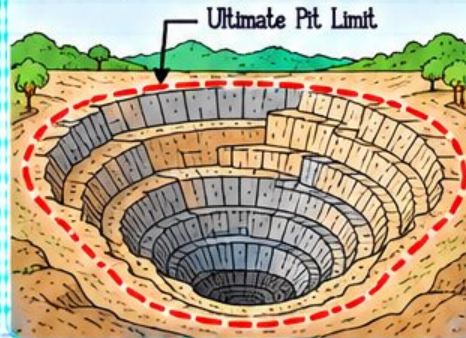
- Ultimate pit design defines the final pit limits to extract maximum economic value from a deposit based on geological, technical and economic considerations.
- It determines the size, shape, depth and final slopes of the pit.

Objective:

- ✓ Maximise Net Present Value (NPV)
- ✓ Define economically viable pit limits
- ✓ Balance ore extraction and waste removal
- ✓ Ensure safe and stable pit slopes
- ✓ Optimise mine life and production schedule

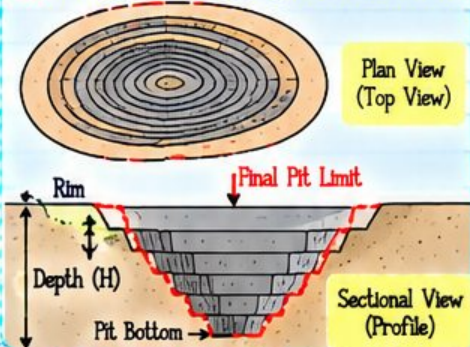
2. Ultimate Pit Concept

- The ultimate pit is the largest economically viable pit.
- Ore blocks inside the final pit limits have positive economic value.



3. Open-Pit Geometry

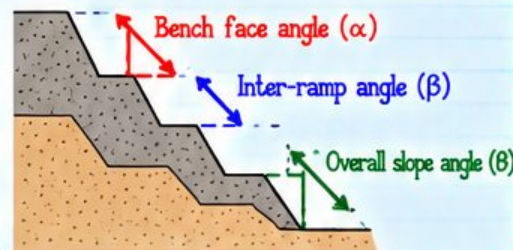
- Shape is generally a conical or irregular bowl depending on orebody geometry, topography and rock strength.
- Final pit is defined in plan, section and 3D.



4. Slope Considerations

- Pit wall slopes depend on rock mass properties, discontinuities, groundwater and in-situ stresses.
- Overall slope angle must be safe and economically feasible.

Slope Type	Typical Angle (°)	Factors
Bench face angle (α)	60 - 80	Rock strength, blasting
Inter-ramp angle (β)	45 - 60	Rock mass, structure
Overall slope angle (θ)	25 - 45	Geometry, height, stability



5. Bench Parameters

- Bench height (H_b): vertical height of each bench
- Bench width (B): horizontal width for drilling, blasting, loading and haulage
- Bench slope: angle of bench face
- Overall bench configuration affects stability and productivity

Parameter	Typical Range
Bench height (H_b)	5 - 15 m
Bench width (B)	6 - 12 m
Bench face angle	60 - 80°
Inter-ramp width	8 - 15 m



6. Stripping Ratio (SR)

- Stripping ratio = quantity of waste removed per unit of ore.
- Higher SR → higher cost
- Lower SR → more economic

$$\text{Stripping Ratio (SR)} = \frac{\text{Tonnage of Waste (W)}}{\text{Tonnage of Ore (O)}}$$

Example:

Parameter	Value
Ore tonnage (O)	50 million tonnes
Waste tonnage (W)	200 million tonnes
$SR = \frac{W}{O} = \frac{200}{50} = 4 : 1$	

Significance:

- ✓ Used to evaluate economic viability
- ✓ Affects mine life and total cost
- ✓ Influences ultimate pit limits

7. Determination of Pit Limits

- Pit limits are determined by economic analysis using:
 - ✓ Cut-off grade
 - ✓ Metal price
 - ✓ Mining cost
 - ✓ Processing cost
 - ✓ Slope constraints
 - ✓ Discount rate (time value of money)

Concept of Economic Pit Limits



8. Methods for Ultimate Pit Design

1. Lerchs-Grossmann (LG) Algorithm (most common)
2. Floating Cone Method
3. Break-even Stripping Ratio Method
4. Whittle Optimization (commercial software)
5. Nested Pit / Pushback Analysis

Pushback Concept:

- Ultimate pit is divided into smaller, economically viable pits (pushbacks) for phased mining.
- Helps in scheduling, cash flow and risk reduction.
- Each pushback is mined sequentially.



9. Design Principles

- ✓ Maximise NPV and economic value.
- ✓ Ensure overall slope stability.
- ✓ Consider geotechnical, hydrogeological and environmental constraints.
- ✓ Optimise pit depth, width and shape.
- ✓ Integrate with production scheduling, infrastructure and processing capacity.
- ✓ Use modern optimisation tools and software (Whittle, MineSight, etc.).

10. Key Takeaways

- ✓ Ultimate pit defines final economic limits of open-pit mining.
- ✓ Slope angles, bench parameters and SR are critical design factors.
- ✓ Larger pit → higher reserves but higher waste and cost.
- ✓ Pit design must balance geology, economics, stability and environment.
- ✓ Pushback and phased mining help in optimising cash flow and reducing risk.

11. Common Mistakes

- ✗ Ignoring slope stability.
- ✗ Using unrealistic cost and metal price.
- ✗ Not considering geotechnical and water conditions.
- ✗ Ignoring phased pushback mining.
- ✗ Overestimating reserves and underestimating costs.

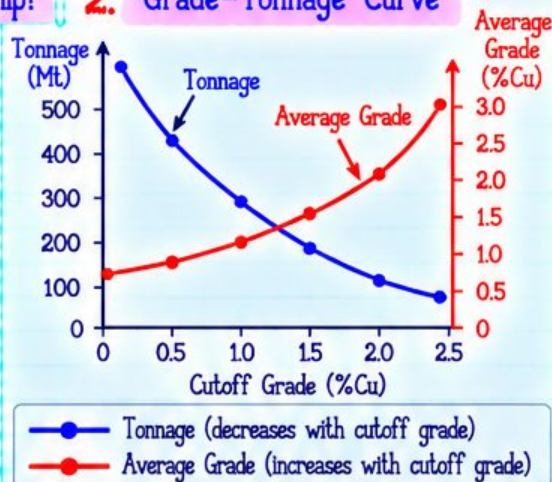
Grade-Tonnage Relationship

"Grade-Tonnage Curves, Cutoff Grade, Average Grade, Marginal Grade, Economic Implications, Curve Interpretation"

1. What is Grade-Tonnage Relationship?

- Shows the relationship between the amount of ore (tonnage) and its average grade for different cutoff grades.
- As cutoff grade increases, tonnage decreases but average grade increases.
- Helps in evaluating economic potential of a deposit and selection of optimal cutoff grade for mine planning.

2. Grade-Tonnage Curve



3. Key Terminologies

- Cutoff Grade (G_c)**
Minimum grade considered economic for extraction.
- Average Grade (G_{avg})**
Weighted average grade of ore above the cutoff grade.
- Marginal Grade (G_m)**
Grade at which revenue equals total mining + processing cost.
- Tonnage (T)**
Total amount of ore above the cutoff grade.
- Grade-Tonnage Curve**
Graph showing variation of tonnage and average grade with different cutoff grades.

4. Effect of Cutoff Grade

Cutoff Grade (%Cu)	Tonnage (Mt)	Average Grade (%Cu)
0.0	500	0.60
0.5	400	0.80
1.0	300	1.10
1.5	200	1.50
2.0	120	2.00
2.5	60	2.50

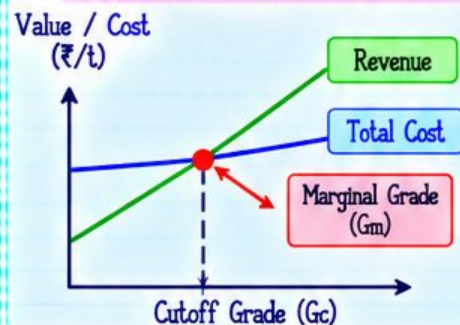
Observations:

- Higher cutoff grade → lower tonnage.
- Higher cutoff grade → higher average grade.
- Selection of optimum cutoff grade balances tonnage and grade for maximum profit.

5. Economic Implications

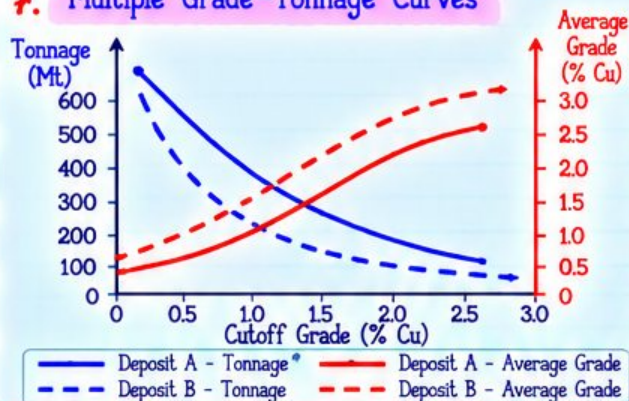
- Lower cutoff grade:**
 - ✓ More tonnage, lower average grade.
 - ✓ Higher mining cost and waste handling.
- Higher cutoff grade:**
 - ✓ Less tonnage, higher average grade.
 - ✓ Lower waste handling, higher unit value.
- Optimal cutoff grade:**
 - ✓ Maximizes Net Present Value (NPV).
 - ✓ Depends on metal price, mining cost, processing cost and recovery.

6. Grade-Tonnage and Profit



At Marginal Grade (G_m) :
 Revenue = Total Cost
 Above G_m → Profitable
 Below G_m → Not Profitable

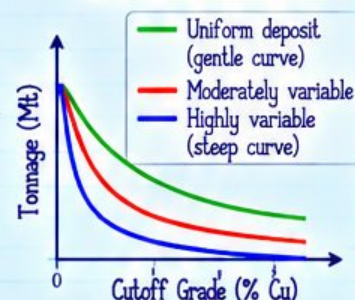
7. Multiple Grade-Tonnage Curves



8. Interpretation of Grade-Tonnage Curve

- Shape of curve indicates uniformity or variability of grade distribution.
- Steep curve → rapidly decreasing tonnage (heterogeneous deposit).
- Gentle curve → gradual decrease in tonnage (more uniform deposit).
- Helps in comparing different deposits.
- Useful for pit design, production planning and economic evaluation.

Typical Deposit Types



9. Grade-Tonnage vs. Metal Content

Cutoff Grade (%Cu)	Tonnage (Mt)	Average Grade (%Cu)	Metal Content (Mt Cu)
0.5	400	0.8	3.20
1.0	300	1.1	3.30
1.5	200	1.5	3.00
2.0	120	2.0	2.40

$$\text{Metal Content (M)} = \text{Tonnage (T)} \times \text{Grade (G)}$$

$$M = T \times G$$

Metal content may increase initially and then decrease with higher cutoff grade.

10. Key Takeaways

- ✓ Grade-tonnage curve shows trade-off between tonnage and grade.
- ✓ Cutoff grade selection is critical for economic viability.
- ✓ Higher cutoff grade → higher average grade but lower tonnage.
- ✓ Marginal grade is the break-even grade.
- ✓ Used for pit design, reserve estimation and production planning.

11. Common Mistakes

- ✗ Ignoring effect of cutoff grade on tonnage and grade.
- ✗ Using single cutoff grade without sensitivity analysis.
- ✗ Not considering metal price and cost variations.
- ✗ Incorrect interpretation of curve shape.
- ✗ Overestimating reserves at low cutoff grade.

Dilution, Recovery & Cut-off Grade

"Ore Dilution, Mining Recovery, Processing Recovery, Cutoff-Grade Concepts, Economic Cutoff Calculations, GATE Numericals"

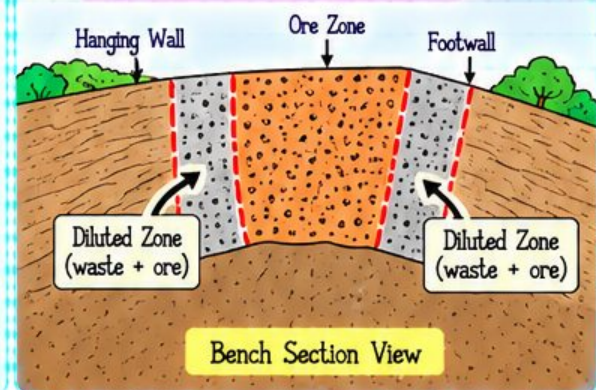
1. What is Dilution?

- ➔ Dilution is the unintended extraction of waste material along with ore.
- ➔ It reduces the grade of the mined material.
- ➔ Caused by imperfect mining methods, geological irregularities, and operational limitations.
- ➔ Measured as dilution % by tonnage or volume.

2. Types of Dilution

- Wall Dilution**
Waste from hanging wall and footwall.
- Internal Dilution**
Waste within the orebody due to low grade zones.
- Operational Dilution**
Due to drilling, blasting, loading and equipment limitations.
- Geological Dilution**
Due to faults, joints, folds and irregular ore boundaries.

3. Illustration of Dilution



4. Dilution Calculations

$$\text{Dilution \% (by tonnage)} = \frac{W}{O + W} \times 100$$

Where,
O = Ore tonnage
W = Waste tonnage (dilution)

Example:

Ore tonnage (O) = 80,000 t
Waste tonnage (W) = 20,000 t

$$\text{Dilution \%} = \frac{20,000}{80,000 + 20,000} \times 100 = 20\%$$

5. What is Recovery?

- ➔ Recovery is the percentage of valuable mineral extracted or recovered from the ore.
- ➔ Two types:

- Mining Recovery**
Portion of ore actually extracted from the deposit.
- Processing Recovery**
Portion of valuable mineral recovered from extracted ore through beneficiation.

6. Recovery Calculations

$$\text{Mining Recovery \%} = \frac{\text{Ore mined}}{\text{In-situ ore}} \times 100$$

$$\text{Processing Recovery \%} = \frac{\text{Metal recovered}}{\text{Metal in feed}} \times 100$$

Example:

In-situ ore = 1,000,000 t
Ore mined = 900,000 t

$$\text{Mining Recovery \%} = \frac{900,000}{1,000,000} \times 100$$

If metal in feed = 10,000 t
Metal recovered = 8,500 t

$$\text{Processing Recovery \%} = \frac{8,500}{10,000} \times 100 = 85\%$$

7. Effect of Dilution and Recovery on Grade

$$\text{Delivered Grade } (G_d) = G_o \times (1 - D) \times R_m$$

Where,
G_o = Original ore grade
D = Dilution fraction (e.g., 0.2 for 20%)
R_m = Mining recovery (fraction)
G_d = Delivered grade to plant

Example:

Original grade (G_o) = 2.0% Cu
Dilution (D) = 20% (0.2)
Mining recovery (R_m) = 90% (0.9)

$$G_d = 2.0 \times (1 - 0.2) \times 0.9 = 2.0 \times 0.8 \times 0.9 = 1.44\% \text{ Cu}$$

8. What is Cut-off Grade?

- ➔ Cut-off grade (G_c) is the minimum grade of ore that can be mined economically.
- ➔ Ore above G_c is considered economic; below G_c is waste.
- ➔ Depends on metal price, mining cost, processing cost and recovery.

Basic Concept

- ✓ If Grade ≥ G_c → Mine as ore
- ✓ If Grade < G_c → Leave as waste

9. Economic Cut-off Grade

$$G_c = \frac{C_m + C_p + C_g + C_s}{(P \times R \times \text{MCF})}$$

C_m = Mining cost (₹/t)
C_p = Processing cost (₹/t)
C_g = General & administrative cost (₹/t)
C_s = Selling cost (₹/t)
P = Metal price (₹/t)
R = Overall recovery (mining × processing)
MCF = Metal conversion factor (to % grade)

Example:

C_m = 600, C_p = 400, C_g = 200, C_s = 100 (₹/t)
P = 500,000 (₹/t), R = 0.85, MCF = 0.01

$$G_c = \frac{600 + 400 + 200 + 100}{500,000 \times 0.85 \times 0.01} = \frac{1,300}{4,250} = 0.31\%$$

10. Overall Flow of Ore to Metal



11. GATE Level Numerical

Q. An ore deposit has grade = 1.5% Cu. Dilution = 15%; Mining recovery = 90%. Processing recovery = 85%.

- Find delivered grade to plant.
- Find final recovered metal grade.

Solution:

- $G_d = 1.5 \times (1 - 0.15) \times 0.90 = 1.5 \times 0.85 \times 0.90 = 1.1475\% \text{ Cu}$
- Final recovered grade = $1.1475 \times 0.85 = 0.975\% \text{ Cu}$

12. Key Takeaways

- ✓ Dilution reduces ore grade and affects economics.
- ✓ Mining and processing recovery determine the final metal output.
- ✓ Cut-off grade is the minimum economic grade for mining.
- ✓ Higher metal price and recovery → lower cut-off grade.
- ✓ Careful control of dilution and improvement in recovery increases project profitability.
- ✓ Important for mine planning, pit design and production scheduling.

Equipment Selection

"Equipment Selection Criteria, Production Capacity, Matching of Loading & Hauling Equipment, Compatibility, Utilization and Operating Factors"

1. Why Equipment Selection is Important?

- ➔ Affects production, cost and mine life.
- ➔ Ensures optimum utilisation of resources.
- ➔ Influences safety, environment and flexibility.
- ➔ Wrong selection leads to higher cost, low productivity and operational problems.

2. Factors Affecting Equipment Selection

- 1 Nature of deposit (ore/waste, hardness, abrasiveness)
- 2 Production capacity required
- 3 Mining method (open-cast / underground)
- 4 Material characteristics (size, density, moisture)
- 5 Haulage distance and profile (flat / gradient)
- 6 Stripping ratio
- 7 Flexibility and future expansion
- 8 Availability of equipment and spares
- 9 Operating and maintenance cost
- 10 Environmental and safety considerations

3. Types of Mining Equipment

- 1 **Loading Equipment**
Shovels, excavators, wheel loaders
- 2 **Hauling Equipment**
Dumps, trucks, belt conveyors
- 3 **Drilling Equipment**
Rotary drills, DTH drills
- 4 **Dozing and Grading Equipment**
Dozers, graders
- 5 **Auxiliary Equipment**
Water tankers, crushers, screens, conveyors

4. Criteria for Equipment Selection

- ✓ Required production capacity (tonnes/hour)
- ✓ Suitability to ground conditions
- ✓ Equipment size and efficiency
- ✓ Compatibility between loading and hauling units
- ✓ Economic evaluation (initial and operating cost)
- ✓ Reliability and availability
- ✓ Ease of maintenance and service support
- ✓ Safety and environmental impact

Key Point: Selection should ensure minimum cost per tonne with maximum productivity and reliability.

5. Production Capacity of Equipment

General Formula :

$$Q = \frac{(q \times n \times E \times U)}{C}$$

- Q = Production capacity (tonnes/hour)
q = Rated bucket/load capacity (m³ or tonnes)
n = Number of cycles per hour
E = Operating efficiency
U = Utilization factor
C = Swell factor (for volume to weight conversion, if required)

Cycle Time (T) :

$$T = t_h + t_s + t_m + t_r$$

- t_h = Hauling time (loaded)
t_s = Spotting and dumping time
t_m = Maneuvering time
t_r = Returning time (empty)

Cycles per hour :

$$n = \frac{60}{T}$$

Example :

Excavator bucket capacity = 3 m³ (≈ 5 t)
Cycle time = 0.5 min
E = 0.9, U = 0.8

$$Q = 5 \times \left(\frac{60}{0.5}\right) \times 0.9 \times 0.8 = 432 \text{ t/h}$$

6. Loading Equipment

Front Shovel



- ➔ Suited for hard rock
- ➔ Higher production rate
- ➔ Used in large open-cast mines

Backhoe



- ➔ Suited for soft to medium hard material
- ➔ Suitable for smaller operations

Wheel Loader



- ➔ Used for short haulage distance
- ➔ High mobility
- ➔ Widely used in surface mining

7. Hauling Equipment

Rigid Dump Truck (RDT)



- ➔ High payload capacity
- ➔ Used in large mines
- ➔ Better for long haul distances

Articulated Dump Truck (ADT)



- ➔ Used for rough terrain
- ➔ More maneuverable
- ➔ Lower initial cost

8. Other Equipment



Dozer

- ➔ Used for spreading, grading and pushback operations
- ➔ Supports mine planning and maintenance



Drill

- ➔ Creates boreholes for blasting
- ➔ Critical for fragmentation and productivity

9. Matching of Loading & Hauling Equipment

Excavator Bucket Size (m ³)	No. of Passes for 60 t Truck	Truck Payload (t)	Recommended Application
3	4 - 5	30 - 40	Small to medium mines
5	3 - 4	40 - 60	Medium mines
8	2 - 3	60 - 90	Large mines
12	2	100 - 150	Very large mines

Key Considerations:

- ➔ Loading time should be minimum (2-5 minutes per truck).
- ➔ Ensure optimum match between bucket size and truck capacity.
- ➔ Avoid underloading or overloading.
- ➔ Consider material fragmentation and swell.

10. Utilization and Operating Factors

- ✓ Utilization factor = Actual working time / Available time
- ✓ Affected by breakdowns, delays, shift change, refuelling.
- ✓ Operating Factors include:
 - Weather conditions
 - Road conditions
 - Maintenance schedules
 - Operator skill and experience
 - Availability of fuel, spares and service support

11. Economic Comparison

Equipment	Initial Cost	Operating Cost	Production Rate	Suitability
Front Shovel	High	Moderate	High	Large, hard rock mines
Backhoe	Moderate	Moderate	Medium	Small/medium mines
Wheel Loader	Moderate	Moderate	Medium	Short haul, auxiliary work
Rigid Dump Truck	High	Low (per tonne)	Very high	Large open-cast mines
Articulated Dump Truck	Moderate	Moderate	Medium	Rough terrain, flexible use

12. Key Takeaways

- ✓ Equipment selection depends on geological, technical, economic and operational factors.
- ✓ Matching of loading and hauling equipment is crucial for maximum productivity.
- ✓ Utilization and operating factors affect actual output.
- ✓ Optimum selection reduces cost per tonne and increases mine economic viability.
- ✓ Use a systematic approach with calculations and field conditions.

25 GATE Numericals + Formula/Concept Revision

"Mineral economics + Sampling + Reserves + Geostatistics + Mine life + Pit design + Grade-tonnage + Cutoff + Equipment selection"

1. Important Formulae (Quick Revision)

- 1 $NPV = \sum_{t=0}^n \frac{(B_t - C_t)}{(1+r)^t}$
- 2 Dilution % = $\frac{W}{O+W} \times 100$
- 3 Delivered grade: $G_d = G_o (1 - D) \times R_m$
- 4 Mining Recovery % = $\frac{\text{Ore mined}}{\text{Ore in place}} \times 100$
- 5 Processing Recovery % = $\frac{\text{Metal recovered}}{\text{Metal in feed}} \times 100$
- 6 Cut-off grade: $G_c = \frac{C_m + C_p + C_g + C_s}{P \times R \times MCF}$
- 7 Mine life (years) = $\frac{\text{Total reserves (t)}}{\text{Annual production (t/y)}}$
- 8 Production rate: $Q = \frac{q \times n \times E \times U}{C}$
- 9 Stripping ratio: $SR = \frac{W}{O}$
- 10 Grade-tonnage (simple):
 $G_{avg} = \frac{\sum (G_i \times T_i)}{\sum T_i}$
- 11 Mineable reserves = Geological reserves $\times$ Mining recovery $\times$ (1 - Dilution)
- 12 Equipment cycle time: $T = t_h + t_s + t_m + t_r$
- 13 Hourly production: $Q = \frac{q \times 60 \times E \times U}{T}$

2. Key Concepts (Quick Revision)

- ✓ Mineral economics: cost, revenue, NPV, payback.
- ✓ Sampling: sample size, error estimation.
- ✓ Reserves: geological vs mineable reserves.
- ✓ Geostatistics: variogram, nugget, sill, range.
- ✓ Mine life: based on reserves and production rate.
- ✓ Pit design: ultimate pit, slope, benches, SR.
- ✓ Grade-tonnage: cutoff grade and curve interpretation.
- ✓ Dilution & recovery: impact on grade and metal.
- ✓ Equipment selection: production capacity, matching, utilization and operating factors.

Data (use where needed, if not given):

- ⓐ Ore density = 2.6 t/m³
- ⓑ Waste density = 2.4 t/m³
- ⓒ Availability = 0.85, Utilization = 0.83
- ⓓ Metal price = ₹5,00,000 / t
- ⓔ Mining cost = ₹400/t, Processing cost = ₹300/t
- ⓕ General cost = ₹100/t, Selling cost = ₹50 / t
- ⓖ MCF = 0.01 (1% grade to metal)

1. Mineral Economics (Q1 - Q5)

- 1 A project requires an initial investment of ₹100 crore. Annual net cash inflow is ₹25 crore for 6 years. Discount rate = 10%. Find NPV.
Ans: ₹3.79 crore
- 2 Mining cost = ₹350/t, Processing cost = ₹250/t, General = ₹100/t, Selling = ₹50/t. Metal price = ₹4,00,000/t, Recovery = 85%, MCF = 0.01. Find cutoff grade.
Ans: 0.22% Cu
- 3 Annual production = 2 million tonnes, Revenue = ₹600 crore/year. Operating cost = ₹400 crore/year. Project life = 10 years. Discount rate = 10%. Find payback period.
Ans: 5 years
- 4 Initial investment = ₹150 crore, annual benefit = ₹40 crore for 8 years, discount rate = 12%. Find NPV.
Ans: ₹32.36 crore
- 5 Metal price increases from ₹4,00,000/t to ₹5,00,000/t. If cutoff grade was 0.25%, new cutoff grade becomes? (Other costs same).
Ans: 0.20% Cu

5. Pit Design (Q14 - Q16)

- 14 A circular pit has top diameter 800 m, bottom diameter 200 m and depth 300 m. Find the volume of material removed.
Ans: 52.36 million m³
- 15 Bench height = 10 m, overall pit slope angle = 45°, find horizontal distance from top to bottom (one side) for depth 300 m.
Ans: 300 m
- 16 Waste volume = 120 million m³, ore volume = 40 million m³, waste density = 2.4 t/m³, ore density = 2.6 t/m³. Find stripping ratio (on weight basis).
Ans: 1.11 : 1

2. Sampling & Estimation (Q6 - Q8)

- 6 A sample of 25 drill holes gives an average grade of 1.2% with standard deviation 0.4%. Find the 95% confidence interval of the mean grade.
Ans: 1.04% to 1.36%
- 7 A rectangular block (100 m $\times$ 50 m $\times$ 20 m) has an average grade of 1.5% Cu. Density = 2.6 t/m³. Estimate the tonnage and metal content.
Ans: 260,000 t, 3,900 t Cu
- 8 Two samples give grades 1.8% and 1.2% with tonnages 500,000 t and 300,000 t. Find the combined average grade.
Ans: 1.58%

6. Grade-Tonnage (Q17 - Q19)

- 17 The table shows grade-tonnage data for a deposit. Find the average grade and total tonnage.
- | Cutoff Grade (%) | Tonnage (Mt) | Grade (%) |
|------------------|--------------|-----------|
| 0.2 | 500 | 0.8 |
| 0.4 | 400 | 1.0 |
| 0.6 | 300 | 1.3 |
| 0.8 | 200 | 1.7 |
- Ans: 350 Mt, 1.20%**
- 18 At cutoff grade 0.5%, tonnage = 250 Mt, average grade = 1.4%. At cutoff 1.0%, tonnage = 120 Mt, average grade = 2.0%. Estimate tonnage at cutoff 0.8% by linear interpolation.
Ans: 170 Mt
 - 19 A grade-tonnage curve shows that at cutoff 0.3% tonnage = 600 Mt and at 0.7% tonnage = 300 Mt. Estimate tonnage at 0.5%.
Ans: 450 Mt

3. Reserves & Mine Life (Q9 - Q11)

- 9 Geological reserves = 120 million tonnes. Mining recovery = 90%. Dilution = 15%. Find mineable reserves.
Ans: 91.8 million tonnes
- 10 Mineable reserves = 80 million tonnes. Annual production = 5 million tonnes/year. Find mine life.
Ans: 16 years
- 11 A mine has proved reserves of 60 million tonnes and probable reserves of 30 million tonnes. Annual production = 6 million tonnes/year. Find total mine life.
Ans: 15 years

7. Dilution, Recovery & Cut-off (Q20 - Q22)

- 20 Original grade = 2.0% Cu, dilution = 20%, mining recovery = 90%. Find delivered grade to plant.
Ans: 1.44% Cu
- 21 Metal in feed = 10,000 t, metal recovered = 8,500 t. Find processing recovery %.
Ans: 85%
- 22 Mining cost = ₹500/t, processing cost = ₹300/t, general = ₹100/t, selling = ₹50/t, metal price = ₹5,00,000/t, recovery = 90%, MCF = 0.01. Find cutoff grade.
Ans: 0.21% Cu

4. Geostatistics (Q12 - Q13)

- 12 A spherical variogram has nugget = 0.1, sill = 1.0 and range = 600 m. Find the semivariance at 300 m.
Ans: 0.55
- 13 A deposit has grades at two locations separated by 200 m with semivariance 0.2. Nugget = 0.1, sill = 0.9, range = 500 m. Assuming spherical model, find the correlation coefficient at 200 m.
Ans: 0.78

8. Equipment Selection (Q23 - Q25)

- 23 A 5 m³ excavator with fill factor 0.9, cycle time 0.5 min, availability 0.85 and utilization 0.80. Find hourly production.
Ans: 367 m³/h (= 954 t/h)
- 24 A truck of 60 t payload has cycle time = 30 min, availability = 0.83 and utilization = 0.80. Find hourly production (tonnes/hour).
Ans: 79.7 t/h
- 25 Match excavator and truck. Excavator production = 900 t/h, truck payload = 45 t, truck cycle time = 25 min, availability = 0.85, utilization = 0.80. Find required number of trucks.
Ans: 12 trucks

Common Data for Practice (Use when not given)

- ✓ Ore density = 2.6 t/m³
- ✓ Waste density = 2.4 t/m³
- ✓ Availability = 0.85
- ✓ Utilization = 0.83
- ✓ Metal price = ₹5,00,000/t
- ✓ Mining cost = ₹400/t
- ✓ Processing cost = ₹300/t
- ✓ General cost = ₹100/t
- ✓ Selling cost = ₹50/t
- ✓ MCF = 0.01 (1% grade to metal)

Key Takeaways

- ✓ Understand and apply important formulae.
- ✓ Be comfortable with unit conversions.
- ✓ Practice grade-tonnage and cutoff calculations.
- ✓ Link dilution, recovery and economics.
- ✓ Solve equipment selection and production problems.
- ✓ These numericals are representative of GATE level.
- ✓ Revise concepts regularly and practice more problems.

Tips to Solve GATE Numericals

- ✓ Write given data and required data clearly.
- ✓ Use consistent units (tonnes, m³, ₹, %).
- ✓ Draw simple diagrams if needed.
- ✓ Apply the correct formula.
- ✓ Check the reasonableness of the answer.
- ✓ Practice regularly under time limit.
- ✓ Focus on conceptual understanding, not just memorising formulae.

Full Mine Planning & Mineral Economics Mock Test

Mixed MCQ + MSQ + NAT, Numerical Problems, Graphs/Curves, Economic Analysis, Reserve Estimation and Mine-Planning Interpretation

1. MCQ - Mineral Economics

Which of the following is a direct cost in mining?

- (A) Depreciation
- (B) Royalty
- (C) Fuel and power
- (D) Administrative salary

2. MCQ - Cut-off Grade

The cut-off grade is primarily used to:

- (A) Maximise mining recovery
- (B) Minimise processing cost only
- (C) Decide whether to mine or leave the ore
- (D) Estimate the total reserves

3. MCQ - Mine Planning

Which planning stage deals with day-to-day operations?

- (A) Strategic planning
- (B) Tactical planning
- (C) Operational planning
- (D) Business planning

4. MCQ - Equipment Selection

Which factor is most important for matching shovel and truck?

- (A) Colour and brand
- (B) Bucket size and truck capacity
- (C) Operator experience
- (D) Maintenance cost

5. MSQ - Advantages of Geostatistics

Which of the following are advantages of geostatistics? (Select all that apply)

- (A) Considers spatial correlation
- (B) Provides estimation variance
- (C) Requires fewer samples
- (D) Generates block grade estimates

6. MSQ - Mine Planning Components

Which of the following are included in mine planning? (Select all that apply)

- (A) Mine layout
- (B) Production scheduling
- (C) Infrastructure planning
- (D) Market demand analysis

7. NAT - Short Answer

Define the following:

- (a) Dilution
- (b) Mining recovery
- (c) Stripping ratio
- (d) Cut-off grade
- (e) Grade-tonnage curve

8. NAT - Short Answer

List any four factors considered in the location of mineral processing plant.

9. NAT - Short Answer

What is the significance of ultimate pit design in open-pit mining?

10. Numerical - Dilution & Recovery

A mine produces 1,00,000 t of ore per year. Dilution = 20% and mining recovery = 85%. If the in-situ grade is 2.0% Cu, find the delivered grade to the plant.

11. Numerical - Economic Cut-off Grade

Metal price = ₹5,00,000/t, mining cost = ₹400/t, processing cost = ₹300/t, general & administrative cost = ₹100/t, selling cost = ₹50/t, and MCF = 0.01 (1% grade to metal). Calculate the economic cut-off grade (in %).

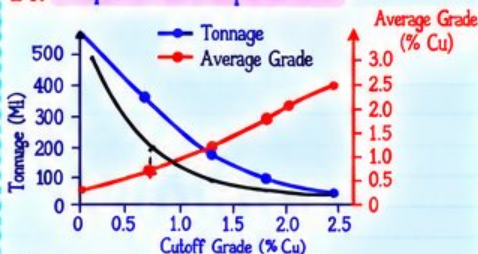
12. Numerical - Mine Life

Proved reserves = 50 million tonnes, annual production = 5 million tonnes, dilution = 10%, mining recovery = 90%. Estimate the mine life.

13. Numerical - Reserve Estimation

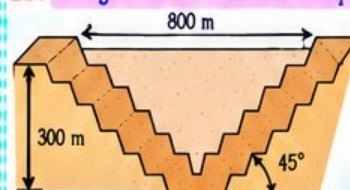
A rectangular block (200 m × 100 m × 50 m) has an average grade of 1.5% Cu. If the density is 2.6 t/m³, estimate the total metal content (tonnes of Cu).

14. Graph/Curve Interpretation



- (a) What is the tonnage at 1.0% Cu?
- (b) What is the average grade at 1.5% Cu?
- (c) Which cutoff grade gives approximately 200 Mt?
- (d) How does average grade change with increasing cutoff grade?

18. Diagram-based Question - Open Pit Design



Given:
Top diameter = 800 m
Bottom diameter = 200 m
Overall depth = 300 m
Overall slope angle = 45°

- (a) Find the volume of material.
- (b) If waste density = 2.4 t/m³, find total waste (tonnes).
- (c) If ore density = 2.6 t/m³ and average grade = 1.2% Cu, estimate total metal (tonnes).

16. Numerical - Equipment Productivity

A shovel has a bucket capacity of 5.0 m³ with fill factor 0.9. Cycle time = 30 s, utilization = 0.80, availability = 0.85. If rock density = 2.4 t/m³, estimate hourly production (tonnes/hour).

17. Numerical - Haulage Cost

A 60 t dump truck hauls material for a one-way distance of 2.5 km at an average speed of 25 km/h (loaded) and 35 km/h (empty). Loading time = 1.0 min, dumping time = 0.5 min. If operating cost = ₹400/t, estimate the total haulage cost per tonne.

19. Numerical - Geostatistics

A spherical variogram model has nugget = 0.1, sill = 1.0 and range = 500 m.

- (a) Write the variogram equation for spherical model.
- (b) Calculate $\gamma(h)$ at $h = 250$ m and $h = 500$ m.

20. Numerical - Grade-Tonnage

For the following block model data:

Cutoff Grade (%)	Tonnage (Mt)	Grade (%)
0.2	600	0.6
0.4	450	0.9
0.6	300	1.2
0.8	180	1.6
1.0	100	2.1

- (a) Plot the grade-tonnage curve (sketch).
- (b) Estimate the average grade at 0.6% cutoff.
- (c) Which cutoff grade gives highest total metal content?

18. Case Study - Mine Planning Interpretation

Annual production schedule for an open-pit mine is given below:

Year	Ore (Mt)	Waste (Mt)	Strip Ratio	Grade (% Cu)
1	5	10	2.0	1.2
2	6	15	2.5	1.1
3	7	21	3.0	1.0
4	8	24	3.0	0.9
5	8	28	3.5	0.8

- (a) Calculate total ore and waste.
- (b) Find average strip ratio for 5 years.
- (c) Comment on the trend of grade with time.

21. NAT - Interpretation

A mine is considering two alternative pit designs:

Parameter	Design A	Design B
Total excavation (Mt)	300	350
Ore (Mt)	80	100
Average grade (% Cu)	1.4	1.0
Strip ratio (W/O)	2.75	2.50

Metal price = ₹5,00,000/t, mining cost = ₹400/t, processing cost = ₹300/t, G & A = ₹100/t, selling cost = ₹50/t, MCF = 0.01.

- (a) Calculate economic value (revenue - total cost) for each design.
- (b) Which design is more economic? Justify.
- (c) What other factors would you consider before final selection?

22. Bonus - Single/Multiple Concept MCQs (5 Questions)

- Q1. Which is NOT a benefit of grade control? (A) Higher dilution (B) Improved plant life (C) Higher recovery (D) Higher grade
- Q2. In open-pit mine, ultimate pit design is to: (A) Maximize ore grade (B) Minimize waste mining (C) Maximize economic value (D) Reduce bench height
- Q3. In geostatistics, the sill represents: (A) Measurement error (B) Total variance (C) Sampling density (D) Range of influence
- Q4. Which is a fixed cost? (A) Drilling (B) Explosives (C) Depreciation (D) Fuel
- Q5. A high strip ratio generally indicates: (A) High-grade ore (B) Low-grade ore (C) Low mining cost (D) Low metal content

Key Takeaways

- ✓ Practice all types: MCQ, MSQ, NAT, numericals, graphs.
- ✓ Focus on formula application and unit conversions.
- ✓ Interpret graphs and tables carefully.
- ✓ Revise concepts of reserves, mine life, pit design, grade-tonnage, cutoff and economics.
- ✓ Develop time management for GATE level.

Self-Check

- ☐ Attempt all questions without notes.
- ☐ Verify answers and calculate step-by-step.
- ☐ Practice similar numericals with different data.
- ☐ Improve speed and accuracy.
- ☐ Revise weak areas from previous days.

Reliability – Fundamentals

Reliability Concept, Failure, Survival Function, Failure Rate, Hazard Rate, MTBF, Reliability Measures

1. What is Reliability?

- Reliability is the probability that a system or component performs its intended function without failure for a specified period of time under stated conditions.
- It measures the dependability and consistency of a system.

Key Point:

Reliability lies between 0 and 1.

- ✓ $R(t) = 1 \rightarrow$ Perfect reliability
- ✓ $R(t) = 0 \rightarrow$ Complete failure

2. Basic Terminology

- Component** : Smallest unit that performs a function.
- System** : Combination of components.
- Failure** : Inability to perform the intended function.
- Time to failure (T)** : Time from start of operation to failure.
- Censored data** : When failure time is not fully observed.

3. Reliability Function $R(t)$

- $R(t)$ is the probability that the component survives up to time t .

$$R(t) = P(T > t)$$

- ✓ $0 \leq R(t) \leq 1$
- ✓ $R(0) = 1$
- ✓ $R(t)$ is a decreasing function of time.

For exponential distribution :

$$R(t) = e^{-\lambda t}$$

4. Failure Distribution Function $F(t)$

- $F(t)$ is the probability that the component fails before time t .

$$F(t) = P(T \leq t)$$

- ✓ $0 \leq F(t) \leq 1$
- ✓ $F(0) = 0, F(\infty) = 1$
- ✓ $R(t) + F(t) = 1$

For exponential distribution :

$$F(t) = 1 - e^{-\lambda t}$$

5. Probability Density Function $f(t)$

- $f(t)$ gives the probability of failure at a particular time t .

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

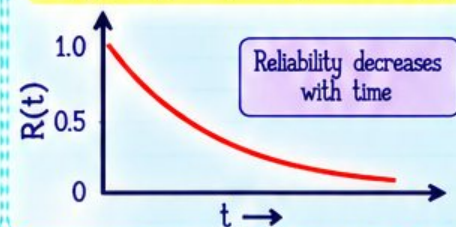
Properties :

- ✓ $f(t) \geq 0$
- ✓ $\int_0^{\infty} f(t)dt = 1$
- ✓ $F(t) = \int_0^t f(x)dx$

6. Survival Function

- Survival function $S(t)$ is same as reliability function $R(t)$.

$$S(t) = P(T > t) = R(t) = 1 - F(t)$$



7. Failure Rate (Hazard Rate) $\lambda(t)$

- Failure rate is the conditional probability of failure in a small time interval, given survival up to time t .

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{P(t \leq T < t + \Delta t | T > t)}{\Delta t}$$

$$\lambda(t) = \frac{f(t)}{R(t)} = -\frac{1}{R(t)} \cdot \frac{dR(t)}{dt}$$

- Unit : failures per unit time (e.g., per hour)
- For exponential distribution, $\lambda(t) = \text{constant} = \lambda$. This is called Constant Failure Rate (CFR).

8. Relationship between $f(t)$, $R(t)$ and $\lambda(t)$

$$f(t) = -\frac{dR(t)}{dt}$$

$$\lambda(t) = \frac{f(t)}{R(t)}$$

$$R(t) = e^{-\int_0^t \lambda(x)dx}$$

$$F(t) = 1 - R(t)$$

9. Mean Time to Failure (MTTF)

- Mean time to failure is the expected value of time to failure.

$$MTTF = E(T) = \int_0^{\infty} tf(t)dt$$

For exponential distribution:

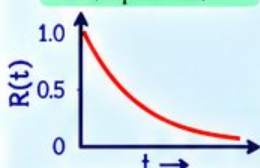
$$MTTF = \frac{1}{\lambda}$$

For general distribution:

$$MTTF = \int_0^{\infty} R(t)dt$$

10. Reliability for Different Failure Rates

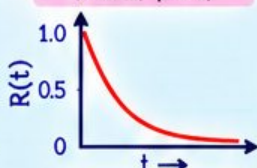
Constant Failure Rate (Exponential)



$$R(t) = e^{-\lambda t}$$

($\lambda = \text{constant}$)

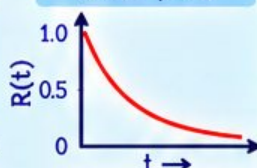
Increasing Failure Rate (Weibull, $\beta > 1$)



$$R(t) = e^{-(t/\eta)^\beta}$$

($\beta > 1$)

Decreasing Failure Rate (Weibull, $\beta < 1$)



$$R(t) = e^{-(t/\eta)^\beta}$$

($\beta < 1$)

11. Reliability Measures (Summary)

Measure	Definition / Formula
Reliability $R(t)$	$P(T > t), 0 \leq R(t) \leq 1$
Failure distribution $F(t)$	$P(T \leq t), F(t) = 1 - R(t)$
Probability density $f(t)$	$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$
Failure rate (Hazard) $\lambda(t)$	$\lambda(t) = \frac{f(t)}{R(t)}$
Survival function $S(t)$	$S(t) = R(t)$
Mean time to failure (MTTF)	$MTTF = \int_0^{\infty} tf(t)dt = \int_0^{\infty} R(t)dt$
Unit	hours, cycles, etc.

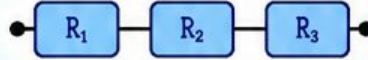
Reliability of Simple Systems

Series Systems, Parallel Systems, Series-Parallel Systems, System Reliability Calculations

1. Series System

Series System (n components)

- ➔ All components are connected in series.
- ➔ System works only if all components work.
- ➔ If any one component fails, system fails.



System Reliability :

$$R_S = R_1 \times R_2 \times R_3 \times \dots \times R_n$$

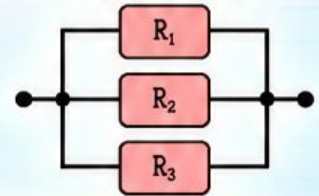
System Failure Probability :

$$F_S = 1 - R_S$$

2. Parallel System

Parallel System (n components)

- ➔ Components are connected in parallel.
- ➔ System works if at least one component works.
- ➔ System fails only if all components fail.



System Reliability :

$$R_P = 1 - (1 - R_1)(1 - R_2) \dots (1 - R_n)$$

For two components :

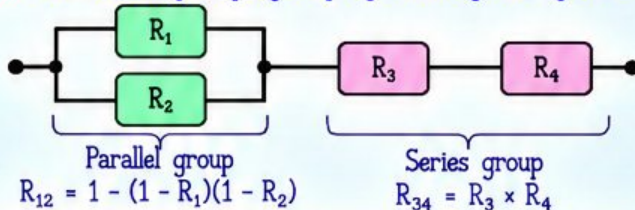
$$R_P = R_1 + R_2 - R_1 R_2$$

System Failure Probability :

$$F_P = (1 - R_1)(1 - R_2) \dots (1 - R_n)$$

3. Mixed Systems (Series-Parallel)

- ➔ Combination of series and parallel configurations.
- ➔ Find reliability step-by-step by reducing the system.



$$R_{12} = 1 - (1 - R_1)(1 - R_2)$$

$$R_{34} = R_3 \times R_4$$

Overall System Reliability :

$$R_{sys} = R_{12} \times R_{34} = [1 - (1 - R_1)(1 - R_2)] \times (R_3 \times R_4)$$

4. k-out-of-n (Redundant) System

- ➔ System works if at least k components work out of n identical components.
- ➔ Used for reliability improvement.

Reliability for n identical components with reliability R :

$$R_{k/n} = \sum_{i=k}^n \binom{n}{i} R^i (1 - R)^{n-i}$$

Special Cases :

- ✓ 1-out-of-n (parallel system) : $R_{1/n} = 1 - (1 - R)^n$
- ✓ n-out-of-n (series system) : $R_{n/n} = R^n$

5. Numerical Example 1 - Series System

A system has three components in series with reliabilities $R_1 = 0.9$, $R_2 = 0.8$ and $R_3 = 0.95$. Find the system reliability and failure probability.

Solution : $R_S = R_1 \times R_2 \times R_3$
 $= 0.9 \times 0.8 \times 0.95$
 $= 0.684$

$$F_S = 1 - R_S = 1 - 0.684 = 0.316$$

Answer :

$$R_S = 0.684$$

$$F_S = 0.316$$

6. Numerical Example 2 - Parallel System

Two components with reliabilities 0.9 and 0.8 are in parallel. Find the system reliability.

Solution : $R_P = R_1 + R_2 - R_1 R_2$
 $= 0.9 + 0.8 - (0.9 \times 0.8)$
 $= 1.7 - 0.72$
 $= 0.98$

$$F_P = 1 - R_P = 1 - 0.98 = 0.02$$

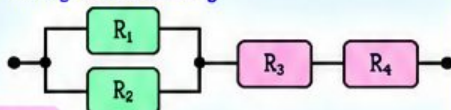
Answer :

$$R_P = 0.98$$

$$F_P = 0.02$$

7. Numerical Example 3 - Series-Parallel System

In the system shown, $R_1 = 0.9$, $R_2 = 0.8$, $R_3 = 0.85$, $R_4 = 0.9$. Find the system reliability.



Solution : $R_{12} = 1 - (1 - 0.9)(1 - 0.8)$
 $= 1 - (0.1 \times 0.2) = 1 - 0.02 = 0.98$

$$R_{34} = 0.85 \times 0.9 = 0.765$$

$$R_{sys} = R_{12} \times R_{34} = 0.98 \times 0.765 = 0.7497$$

Answer :

$$R_{sys} = 0.7497$$

8. Numerical Example 4 - k-out-of-n System

A system has 3 identical components ($R = 0.8$) and works if at least 2 components work. Find the system reliability.

Solution : $R_{2/3} = {}^3C_2 R^2 (1 - R) + {}^3C_3 R^3$
 $= 3 (0.8)^2 (0.2) + 1 (0.8)^3$
 $= 3 (0.64) (0.2) + 0.512$
 $= 0.384 + 0.512 = 0.896$

Answer :

$$R_{2/3} = 0.896$$

For comparison :

$$1\text{-out-of-3} : R_{1/3} = 1 - (1 - 0.8)^3 = 0.992$$

$$3\text{-out-of-3} : R_{3/3} = (0.8)^3 = 0.512$$

9. Key Takeaways

- ✓ Series system : all components must work.
- ✓ Parallel system : at least one component must work.
- ✓ System reliability depends on configuration.
- ✓ Redundancy improves reliability.
- ✓ For mixed systems, reduce step by step.
- ✓ k-out-of-n system is used for redundancy design.

10. Important Formulas (Quick Revision)

- ➔ Series : $R_S = R_1 R_2 \dots R_n$
 $F_S = 1 - R_S$
- ➔ Parallel : $R_P = 1 - (1 - R_1)(1 - R_2) \dots (1 - R_n)$
 $F_P = (1 - R_1)(1 - R_2) \dots (1 - R_n)$
- ➔ Two parallel : $R_P = R_1 + R_2 - R_1 R_2$
- ➔ k-out-of-n : $R_{k/n} = \sum_{i=k}^n \binom{n}{i} R^i (1 - R)^{n-i}$

Applications in Mining

- ✓ Reliability of mining equipment systems.
- ✓ Design of redundant systems for critical operations.
- ✓ Planning maintenance based on reliability analysis.
- ✓ Used for availability improvement.

Reliability - Combined Systems & Numericals

Redundancy, Standby Systems, Minimal Paths/Cuts, Block Diagrams, GATE-Level Reliability Problems

1. Redundancy in Reliability

→ Redundancy means using extra components to improve system reliability.

→ Types :

- ① Active redundancy (parallel)
- ② Standby redundancy
- ③ k-out-of-n redundancy

2. Standby Systems

→ Standby component comes into operation only after the main component fails.

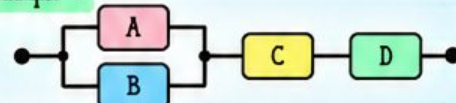
→ Types :

- ① Cold standby - perfect switching, no failure while in standby.
- ② Warm standby - some failure while in standby.
- ③ Hot standby - components are always in operation (parallel).

3. Minimal Path and Minimal Cut Sets

→ Minimal path set : Smallest set of components that ensures system works.
→ Minimal cut set : Smallest set of components whose failure causes system failure.

Example :



Minimal path sets : {A,C,D} {B,C,D}
Minimal cut sets : {C} {D} {A,B}

4. k-out-of-n System

→ System works if at least k out of n components work.

Reliability :

$$R_{k/n} = \sum_{i=k}^n \binom{n}{i} R^i (1-R)^{n-i}$$

5. Standby System Reliability

(a) Two-unit cold standby system



$$R(t) = R_M(t) + [1 - R_M(t)] R_S(t)$$

For exponential distribution ($\lambda_M = \lambda_S = \lambda$):

$$R(t) = e^{-\lambda t} + (1 - e^{-\lambda t}) e^{-\lambda t} = 2e^{-\lambda t} - e^{-2\lambda t}$$

6. Warm and Hot Standby

(a) Warm standby (standby failure rate = λ_s)

$$R(t) = e^{-\lambda t} + \frac{\lambda_s}{\lambda_s - \lambda} (e^{-\lambda t} - e^{-\lambda_s t})$$

(b) Hot standby (active redundancy)
Equivalent to parallel system.

$$R(t) = 1 - [1 - e^{-\lambda t}]^2 = 2e^{-\lambda t} - e^{-2\lambda t}$$

7. Block Diagram Reduction Steps

- ① Identify series/parallel groups.
- ② Reduce step-by-step using formulas.
- ③ Use minimal path/cut sets for complex systems.
- ④ Draw equivalent simplified diagram.
- ⑤ Calculate system reliability.

8. Example - Series-Parallel Combination



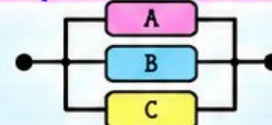
$R_A = 0.9, R_B = 0.8, R_C = 0.95$
Find system reliability.

Solution :

$$R_{AB} = 1 - (1 - R_A)(1 - R_B) = 1 - (0.1)(0.2) = 0.98$$

$$R_{sys} = R_{AB} \times R_C = 0.98 \times 0.95 = 0.931$$

9. Example - Three Parallel Units



$R_A = 0.8, R_B = 0.85, R_C = 0.9$
Find system reliability.

Solution :

$$R_p = 1 - (1 - 0.8)(1 - 0.85)(1 - 0.9) = 1 - (0.2)(0.15)(0.1) = 1 - 0.003 = 0.997$$

10. Example - k-out-of-n System

5-out-of-7 identical components,
 $R = 0.9$. Find system reliability.

Solution :

$$R_{5/7} = \sum_{i=5}^7 \binom{7}{i} (0.9)^i (0.1)^{7-i} = 21(0.9)^5(0.1)^2 + 7(0.9)^6(0.1) + (0.9)^7 = 0.10206 + 0.37201 + 0.47830 = 0.95237$$

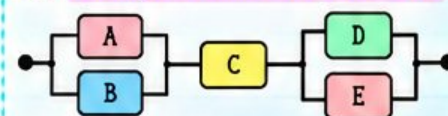
11. Example - Cold Standby System

Two identical units, $\lambda = 0.01$ per hour,
 $t = 100$ hours. Find reliability.

Solution :

$$R(t) = 2e^{-\lambda t} - e^{-2\lambda t} = 2e^{-0.01 \times 100} - e^{-0.02 \times 100} = 2e^{-1} - e^{-2} = 2(0.3679) - 0.1353 = 0.6005$$

12. Example - Minimal Path/Cut Sets



Minimal path sets : {A,C,D} {A,C,E}
{B,C,D} {B,C,E}

Minimal cut sets : {C} {A,B} {D,E}

13. Important Points for GATE

- ✓ Identify minimal path and cut sets for complex systems.
- ✓ Use block diagram reduction step-by-step.
- ✓ k-out-of-n and standby systems are frequently asked.
- ✓ For exponential distribution, reliability calculations are simple.
- ✓ Draw clear and neat block diagrams.
- ✓ Practice previous GATE questions.

14. Key Formulas (Quick Revision)

Case	Reliability Formula
Series	$R_s = \prod_{i=1}^n R_i$
Parallel	$R_p = 1 - \prod_{i=1}^n (1 - R_i)$
k-out-of-n	$R_{k/n} = \sum_{i=k}^n \binom{n}{i} R^i (1-R)^{n-i}$
Cold standby (2 units)	$R(t) = 2e^{-\lambda t} - e^{-2\lambda t}$

15. Applications in Mining

- ✓ Design of redundant mine ventilation fans, pumps and conveyors.
- ✓ Reliability of load-haul-dump (LHD) and drilling systems.
- ✓ Maintenance planning using standby units.
- ✓ Availability improvement of critical equipment.
- ✓ Used in mine safety and continuous production planning.

Maintainability & Availability

Maintainability Concept, Repair Rate, MTTR, Availability, Inherent/Operational Availability, Reliability - Availability Relationships

1. Maintainability - Concept

- ➔ Maintainability is the probability that a failed item can be repaired to a specified condition within a given time under stated conditions.
- ➔ It measures ease and speed of repair.
- ➔ Higher maintainability → lower downtime → higher availability.

Maintainability function :

$$M(t) = P(T_r \leq t)$$

2. Repair Time and Repair Rate

- ➔ Repair time (T_r) : Time required to restore a failed item to working condition.
- ➔ Repair rate (μ) : Rate at which repairs are completed.
- ➔ Assume exponential distribution of repair time.

Repair time distribution :

$$f_r(t) = \mu e^{-\mu t}, t \geq 0$$

Maintainability function :

$$M(t) = 1 - e^{-\mu t}$$

3. Mean Time to Repair (MTTR)

- ➔ Mean Time to Repair (MTTR) is the expected value of repair time.
- ➔ For exponential distribution :

$$MTTR = \int_0^{\infty} t f_r(t) dt = \frac{1}{\mu}$$

- ➔ $\mu = 1 / MTTR$
- ➔ Smaller MTTR → higher availability and maintainability.

4. Availability - Concept

- ➔ Availability is the probability that the item is in operating (up) condition at a random time.
- ➔ It considers both failure and repair.

$$A(t) = \frac{MTBF}{MTBF + MTTR}$$

$$= \frac{\mu}{\lambda + \mu}$$

5. Types of Availability

(a) Inherent Availability (A_i)

- ➔ Considers only active repair time (no logistic or administrative delay).

$$A_i = \frac{MTBF}{MTBF + MTTR} = \frac{\mu}{\lambda + \mu}$$

(b) Operational Availability (A_o)

- ➔ Considers active repair time + logistic delay time (MDT) + administrative delay time (ADT).

$$A_o = \frac{MTBF}{MTBF + MTTR + MDT + ADT}$$

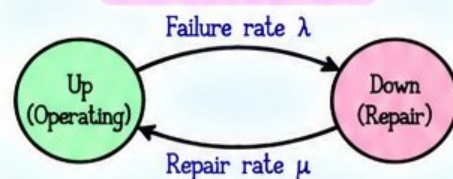
6. Relationship between Reliability, Maintainability and Availability

- ➔ Reliability $R(t)$: Probability of no failure up to time t .
- ➔ Maintainability $M(t)$: Probability of repair within time t after failure.
- ➔ Steady-state availability for exponential distributions :

$$A = \frac{\mu}{\lambda + \mu} = \frac{MTBF}{MTBF + MTTR}$$

- ➔ Higher reliability (lower λ) and higher maintainability (higher μ) lead to higher availability.

7. Reliability-Availability Model (Two-State Model)



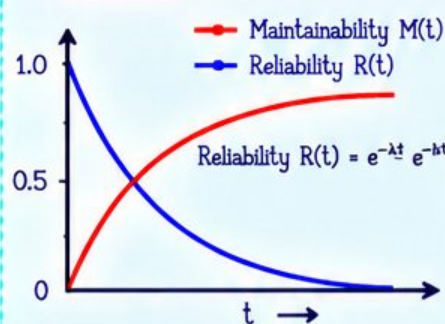
Steady-state availability :

$$A = \frac{\mu}{\lambda + \mu}$$

Steady-state unavailability :

$$U = \frac{\lambda}{\lambda + \mu} = 1 - A$$

8. Reliability and Maintainability Curves



9. Numerical Example 1 - Availability

A machine has MTBF = 500 hours and MTTR = 20 hours. Find (a) Inherent availability (b) Unavailability.

Solution :

$$A = \frac{MTBF}{MTBF + MTTR}$$

$$= \frac{500}{500 + 20}$$

$$= 0.9615$$

$$U = 1 - A = 0.0385$$

Answer :

$$A = 0.9615$$

$$U = 0.0385$$

10. Numerical Example 2 - Operational Availability

MTBF = 400 hours, MTTR = 16 hours, MDT = 8 hours, ADT = 6 hours. Find operational availability.

Solution :

$$A_o = \frac{400}{400 + 16 + 8 + 6}$$

$$= \frac{400}{430}$$

$$= 0.9302$$

Answer :

$$A_o = 0.9302$$

11. Comparison of Availability Types

Type	Definition	Formula
Inherent Availability (A_i)	Considers only active repair time	$A_i = \frac{MTBF}{MTBF + MTTR}$
Operational Availability (A_o)	Considers active repair time + logistic + admin delays	$A_o = \frac{MTBF}{MTBF + MTTR + MDT + ADT}$
Achieved Availability	Includes preventive maintenance and operational factors	$A = \frac{\text{Uptime}}{\text{Total time}}$

12. Key Takeaways

- ✓ Maintainability measures ease and speed of repair.
- ✓ $MTTR = 1/\mu$ for exponential repair time.
- ✓ Availability = $MTBF / (MTBF + MTTR)$.
- ✓ Inherent availability ignores logistic and admin delays.
- ✓ Operational availability includes MDT and ADT.
- ✓ Higher reliability ($\downarrow \lambda$) and higher maintainability ($\uparrow \mu$) improve availability.
- ✓ Use two-state model for GATE problems.

Linear Programming - Fundamentals

LP Formulation, Decision Variables, Objective Function, Constraints, Feasible Region, Graphical Method

1. What is Linear Programming (LP)?

- Linear Programming is a mathematical technique to find the best (optimal) solution (maximum or minimum) of a linear objective function, subject to linear equality or inequality constraints.
- It is widely used for resource allocation, planning and optimization problems.

2. Standard Form of LPP

Maximize / Minimize :

$$Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

Subject to :

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n (\leq, =, \geq) b_i$$

$$i = 1, 2, \dots, m$$

$$x_1, x_2, \dots, x_n \geq 0 \quad (\text{Non-negativity})$$

3. Components of an LPP

- Decision variables** ($x_1, x_2, \dots, x_n$)
Unknown quantities to be determined.
- Objective function (Z)**
Linear function to be maximized or minimized.
- Constraints**
Linear equations or inequalities representing limitations.
- Non-negativity constraints**
All decision variables are ≥ 0 .

4. Formulation Steps

- Identify decision variables.
- Write the objective function (Maximize or Minimize).
- Write all the constraints.
- Add non-negativity constraints.
- Solve using graphical or simplex method.

5. Example - Problem Formulation

A mine uses two types of shovels, A and B. Profit from A = ₹6 per hour and from B = ₹5 per hour. Due to manpower and maintenance constraints:

- Total hours of A and B ≤ 100
- Hours of A ≤ 60
- Hours of B ≤ 80

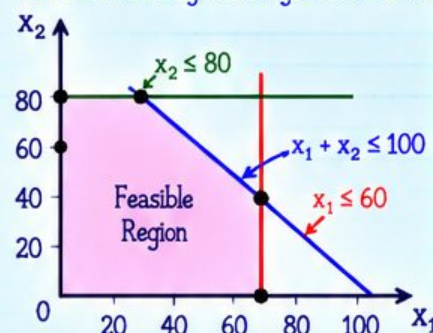
Formulate the LPP.

Solution : Let x_1 = hours of shovel A
 x_2 = hours of shovel B

$$\begin{aligned} \text{Maximize } Z &= 6x_1 + 5x_2 \\ \text{Subject to : } & \\ &x_1 + x_2 \leq 100 \\ &x_1 \leq 60 \\ &x_2 \leq 80 \\ &x_1, x_2 \geq 0 \end{aligned}$$

6. Feasible Region

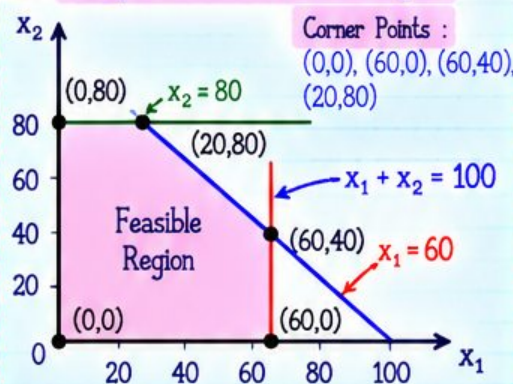
- The feasible region is the set of all points that satisfy all constraints simultaneously.
- It is a convex polygon in two variables and a convex region in higher dimensions.



7. Graphical Method - Steps

- Convert each constraint into an equality and plot the line.
- Identify the feasible region (common region).
- Find the corner points (vertices).
- Evaluate Z at each corner point.
- Choose the point giving maximum or minimum value of Z.

8. Graphical Solution - Example



9. Evaluate Objective Function

$$Z = 6x_1 + 5x_2$$

Corner Point	x_1	x_2	$Z = 6x_1 + 5x_2$
(0,0)	0	0	0
(60,0)	60	0	360
(60,40)	60	40	560
(20,80)	20	80	520

Maximum profit = 560
at $(x_1, x_2) = (60, 40)$

10. Types of Constraints

- $\leq$ type : Region below the line.
- $\geq$ type : Region above the line.
- $=$ type : Points on the line.

11. Key Points to Remember

- ✓ Feasible region is always convex.
- ✓ Optimal solution occurs at a corner point.
- ✓ Number of corner points is finite.
- ✓ If feasible region is unbounded $\rightarrow$ unbounded solution possible.
- ✓ If no common region $\rightarrow$ infeasible solution.
- ✓ If objective function is parallel to an edge $\rightarrow$ multiple optimal solutions.

12. Applications in Mining

- ✓ Production planning.
- ✓ Allocation of manpower and equipment.
- ✓ Blend optimization of ores.
- ✓ Scheduling of mining operations.
- ✓ Resource allocation (shovels, trucks).
- ✓ Cost minimization / profit maximization.
- ✓ Used for short-term and long-term mine planning decisions.

Linear Programming - Solutions & Numericals

Graphical Optimization, Corner-Point Method, Maximization/Minimization, Infeasible/Unbounded Solutions, GATE Numericals

1. Graphical Solution Steps

- 1 Plot each constraint as an equality.
- 2 Identify the feasible region.
- 3 Find corner points (vertices).
- 4 Evaluate the objective function at each corner point.
- 5 Choose the point giving maximum or minimum value of Z .

2. Maximization Problem - Example

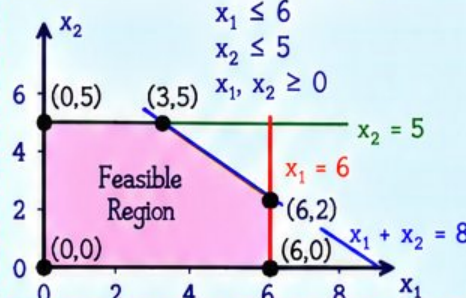
Maximize $Z = 3x_1 + 5x_2$

Subject to : $x_1 + x_2 \leq 8$

$x_1 \leq 6$

$x_2 \leq 5$

$x_1, x_2 \geq 0$



Evaluate Z at corner points :

Point	x_1	x_2	$Z = 3x_1 + 5x_2$
(0,0)	0	0	0
(6,0)	6	0	18
(6,2)	6	2	28
(3,5)	3	5	34
(0,5)	0	5	25

Maximum $Z = 34$
at $(x_1, x_2) = (3, 5)$

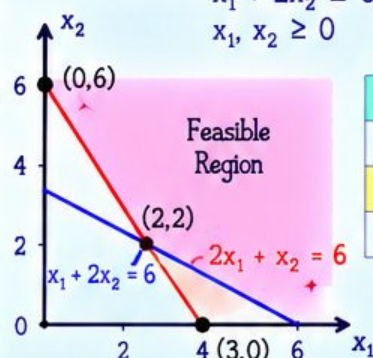
3. Minimization Problem - Example

Minimize $Z = 2x_1 + 3x_2$

Subject to : $2x_1 + x_2 \geq 6$

$x_1 + 2x_2 \geq 6$

$x_1, x_2 \geq 0$



Corner points :

Point	x_1	x_2	$Z = 2x_1 + 3x_2$
(0,6)	0	6	18
(2,2)	2	2	10
(3,0)	3	0	6

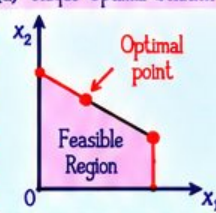
Minimum $Z = 6$
at $(x_1, x_2) = (3, 0)$

4. Corner-Point (Vertex) Method

- 1 Find all corner points of feasible region.
- 2 Substitute each corner point in objective function.
- 3 For maximization $\rightarrow$ choose largest Z .
For minimization $\rightarrow$ choose smallest Z .
- 4 If multiple optimal solutions exist $\rightarrow Z$ is same on all points along an edge.

5. Special Cases

(a) Unique Optimal Solution



(b) Multiple Optimal Solutions



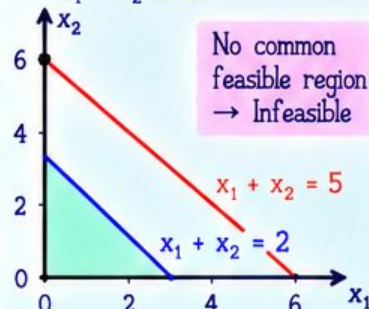
(c) Unbounded Solution



6. Infeasible Solution

$x_1 + x_2 \leq 2$

$x_1 + x_2 \geq 5$



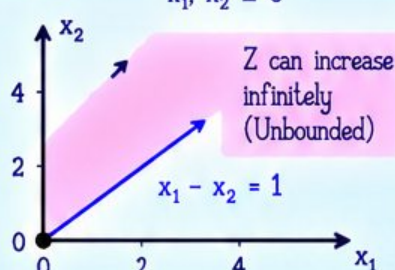
No common feasible region
 $\rightarrow$ Infeasible

7. Unbounded Solution - Example

Maximize $Z = x_1 + x_2$

Subject to : $x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$



Z can increase infinitely
(Unbounded)

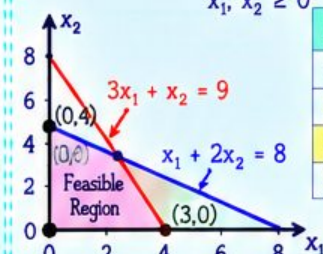
8. GATE Level Numerical Example

Maximize $Z = 4x_1 + 3x_2$

Subject to : $x_1 + 2x_2 \leq 8$

$3x_1 + x_2 \leq 9$

$x_1, x_2 \geq 0$



Point	x_1	x_2	$Z = 4x_1 + 3x_2$
(0,0)	0	0	0
(3,0)	3	0	12
(2,3)	2	3	17
(0,4)	0	4	12

Maximum $Z = 17$
at $(x_1, x_2) = (2, 3)$

9. Key Formulas & Points

- ✓ LP solution always lies at a corner point of the feasible region.
- ✓ For n constraints, number of corner points is finite.
- ✓ Use graphical method only for two variables.
- ✓ Maximization $\rightarrow$ choose highest Z .
- ✓ Minimization $\rightarrow$ choose lowest Z .
- ✓ Check feasibility, boundedness and optimality.

10. Common GATE Questions

- ✓ Find optimal solution for given LP.
- ✓ Identify feasible / infeasible / unbounded.
- ✓ Multiple optimal solutions.
- ✓ Value of Z at corner points.
- ✓ Effect of adding new constraints.
- ✓ Shadow of objective function line.
- ✓ Graph-based reasoning questions.

11. Applications in Mining

- ✓ Optimal allocation of resources (machines, labor, budget).
- ✓ Production planning and scheduling.
- ✓ Blend optimization of ores.
- ✓ Planning of drilling, excavation, transport.
- ✓ Cost minimization and profit maximization.
- ✓ Used for short-term and long-term mine design and planning.

Transportation Problems

Transportation Model, Balanced/Unbalanced Problems, Initial Basic Feasible Solution, Cost Minimization, MODI/Optimality Concepts

1. Transportation Model - Concept

- A transportation problem deals with finding the optimal shipping plan to minimize the total transportation cost from multiple sources (suppliers) to multiple destinations (consumers).
- It is a special type of linear programming problem.
- Assumptions :

- 1 Single product.
- 2 Supply and demand are known.
- 3 Transportation cost is constant.
- 4 Total supply = total demand (for balanced problem).
- 5 All decision variables are ≥ 0 .

2. Mathematical Formulation

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to :

$$\sum_{j=1}^n x_{ij} = s_i \quad (i = 1, 2, \dots, m) \quad (\text{Supply constraints})$$

$$\sum_{i=1}^m x_{ij} = d_j \quad (j = 1, 2, \dots, n) \quad (\text{Demand constraints})$$

$$x_{ij} \geq 0 \quad \text{for all } i, j$$

m = number of sources (suppliers)
 n = number of destinations (consumers)
 c_{ij} = transportation cost from i to j
 x_{ij} = quantity shipped from i to j

3. Types of Problems

(a) Balanced Transportation Problem

- Total supply = Total demand
- No dummy row/column is required.

Example :

From \ To	D ₁	D ₂	D ₃	Supply
S ₁	2	3	5	20
S ₂	4	1	7	30
S ₃	6	5	2	25
Demand	10	30	35	75

Total Supply = 75
 Total Demand = 75
 (Balanced)

(b) Unbalanced Transportation Problem

- Total supply $\neq$ Total demand
- Add a dummy row/column with zero transportation cost to balance the problem.

4. Initial Basic Feasible Solution (IBFS) Methods

- 1 North-West Corner Rule (NWCR)
 - Simplest method
 - Start from top-left cell
- 2 Least Cost Method (LCM)
 - Choose cell with least cost first
- 3 Vogel's Approximation Method (VAM)
 - Usually gives better initial solution
 - Based on penalty (difference of two least costs)

5. Example - Initial Solution using NWCR

Costs (₹ per unit)

From \ To	D ₁	D ₂	D ₃	Supply
S ₁	2	3	5	20
S ₂	4	1	7	30
S ₃	6	5	2	25
Demand	10	30	35	

NWCR Steps :

- Start at (1,1): allocate $\min(20, 10) = 10$
- Move right/down
- Continue until all supply and demand are satisfied.

Initial Basic Feasible Solution (NWCR) :

From \ To	D ₁	D ₂	D ₃	Supply
S ₁	10	10	-	20
S ₂	-	20	10	30
S ₃	-	-	25	25
Demand	10	30	35	

Total Transportation Cost :

$$Z = 2(10) + 3(10) + 1(20) + 7(10) + 2(25) = 20 + 30 + 20 + 70 + 50 = 190$$

6. Example - Initial Solution using LCM

Costs (₹ per unit)

From \ To	D ₁	D ₂	D ₃	Supply
S ₁	2	3	5	20
S ₂	4	1	7	30
S ₃	6	5	2	25
Demand	10	30	35	

LCM Steps :

- 1 Choose least cost cell.
- 2 Allocate maximum possible.
- 3 Cross out satisfied row/column.
- 4 Repeat for next least cost.

Initial Solution (LCM) :

From \ To	D ₁	D ₂	D ₃	Supply
S ₁	10	-	10	20
S ₂	-	30	-	30
S ₃	-	-	2	25
Demand	10	30	35	

Total Cost :

$$Z = 2(10) + 5(10) + 1(30) + 2(25) = 20 + 50 + 30 + 50 = 150$$

7. Vogel's Approximation Method (VAM)

- 1 Calculate penalty for each row and column (difference of two least costs).
- 2 Choose the row/column with highest penalty.
- 3 Allocate to the least cost cell in that row/column.
- 4 Cross out satisfied row/column and repeat.

8. Optimality Test - MODI Method

- 1 Calculate potentials u_i and v_j for all basic cells ($u_i + v_j = c_{ij}$).
- 2 Find opportunity cost (Δ_{ij}) for non-basic cells:

$$\Delta_{ij} = c_{ij} - (u_i + v_j)$$
- 3 If all $\Delta_{ij} \geq 0 \rightarrow$ current solution is optimal (for minimization).
- 4 If any $\Delta_{ij} < 0 \rightarrow$ enter the cell with most negative Δ_{ij} and improve the solution using closed loop method.

9. MODI Method - Example (One Iteration)

From \ To	D ₁	D ₂	D ₃	Supply
S ₁	10	10	-	20
S ₂	-	20	10	30
S ₃	-	-	25	25
Demand	10	30	35	

Step 1 : Find u_i and v_j

$$\begin{aligned} u_1 &= 0 & v_1 &= 2, v_2 = 3 \\ u_2 &= -2 & v_3 &= 9 \\ u_3 &= -7 \end{aligned}$$

Step 2 : Opportunity costs

$$\begin{aligned} \Delta_{13} &= 5 - (0 + 9) = -4 \\ \Delta_{31} &= 6 - (-7 + 2) = 11 \end{aligned}$$

Since $\Delta_{13} < 0$, current solution is not optimal.

Select (1,3) to enter and form a closed loop for improvement.

10. Closed Loop Method

- 1 Form a closed loop through basic cells and the entering cell.
- 2 Mark + and - at alternate corners.
- 3 Find the minimum allocation (θ) at cells marked with -.
- 4 Add θ to + cells and subtract θ from - cells.
- 5 Repeat until all $\Delta_{ij} \geq 0$.

11. Key Points to Remember

- ✓ Total number of basic variables = $m + n - 1$.
- ✓ If less, add a small value (ϵ) to make it non-degenerate.
- ✓ VAM generally gives a better initial solution.
- ✓ MODI method is used for optimality test.
- ✓ For maximization $\rightarrow$ convert to minimization (multiply by -1).

12. Applications in Mining

- ✓ Transportation of coal, ore and minerals.
- ✓ Allocation of mining equipment to different sites.
- ✓ Logistics planning and distribution of resources.
- ✓ Helps in cost minimization and efficient planning.

Assignment Problems

Assignment Formulation, Hungarian Method, Minimization/Maximization, Unbalanced Assignment, Prohibited Assignments, Numericals

1. Assignment Problem - Concept

- ➔ Assignment problem is a special case of transportation problem where each source is assigned to exactly one destination (one-to-one assignment).
- ➔ Objective : To minimize or maximize the total cost/profit of assignment.
- ➔ It is solved using Hungarian method.

Assumptions :

- ✓ Number of jobs = number of persons (square matrix).
- ✓ Each person is assigned to exactly one job.
- ✓ All costs/profits are known and finite.

2. Mathematical Formulation

Minimization form :

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to :

$$\sum_{j=1}^n x_{ij} = 1 \quad (i = 1, 2, \dots, m)$$

$$\sum_{i=1}^m x_{ij} = 1 \quad (j = 1, 2, \dots, n)$$

$$x_{ij} = 0 \text{ or } 1$$

$x_{ij} = 1 \rightarrow$ person i assigned to job j

$x_{ij} = 0 \rightarrow$ otherwise

c_{ij} = cost of assigning person i to job j

3. Types of Assignment Problems

- 1. Minimization Assignment Problem**
Find assignment with minimum total cost.
- 2. Maximization Assignment Problem**
Find assignment with maximum total profit.
- 3. Unbalanced Assignment Problem**
Number of persons $\neq$ number of jobs.
Add dummy row/column with zero cost (min) or zero profit (max).
- 4. Prohibited Assignment**
Some assignments are not allowed.
Assign a very large cost (M) for minimization or a very small profit ($-M$) for maximization.

4. Hungarian Method - Steps (Minimization)

- 1** If it is a maximization problem, convert to minimization (by subtracting each element from the largest element).
- 2** Row reduction : Subtract the smallest element of each row from all elements of that row.
- 3** Column reduction : Subtract the smallest element of each column from all elements of that column.
- 4** Cover all zeros with minimum number of horizontal and vertical lines.
- 5** If number of lines = $n \rightarrow$ optimal assignment exists. Otherwise, find the smallest uncovered element, subtract it from all uncovered elements and add it to elements at intersection of lines. Repeat from Step 4.
- 6** Make assignment (one zero per row and column) and compute total cost/profit.

5. Example 1 - Minimization Assignment Problem

Find the optimal assignment with minimum total cost.

Cost Matrix (₹)

Person	J ₁	J ₂	J ₃	J ₄
P ₁	9	2	7	8
P ₂	6	4	3	7
P ₃	5	8	1	8
P ₄	7	6	9	4

Optimal Assignment :

- P₁ $\rightarrow$ J₂ (2)
- P₂ $\rightarrow$ J₁ (6)
- P₃ $\rightarrow$ J₃ (1)
- P₄ $\rightarrow$ J₄ (4)

Minimum total cost
 $Z = 2 + 6 + 1 + 4$
 $= 13$

6. Example 2 - Maximization Assignment Problem

Find the optimal assignment with maximum total profit.

Profit Matrix (₹)

Person	J ₁	J ₂	J ₄
P ₁	10	14	8
P ₂	9	7	15
P ₃	8	12	6

Optimal Assignment :

- P₁ $\rightarrow$ J₂ (14)
- P₂ $\rightarrow$ J₃ (15)
- P₃ $\rightarrow$ J₁ (8)

Maximum total profit
 $Z = 14 + 15 + 8$
 $= 37$

7. Example 3 - Unbalanced Assignment Problem

4 persons and 3 jobs. Minimize total cost.

Cost Matrix (₹)

Person	J ₁	J ₂	J ₃
P ₁	4	8	7
P ₂	6	5	9
P ₃	7	3	8
P ₄	6	4	5

Add a dummy job
J₄ with zero cost.

Modified Matrix

Person	J ₁	J ₂	J ₃	J ₄
P ₁	4	8	7	0
P ₂	6	5	9	0
P ₃	7	3	8	0
P ₄	6	4	5	0

Optimal Assignment : P₁ $\rightarrow$ J₁, P₂ $\rightarrow$ J₂, P₃ $\rightarrow$ J₃, P₄ $\rightarrow$ J₄ (Dummy)

Minimum total cost $Z = 4 + 5 + 8 + 0 = 17$

8. Example 4 - Prohibited Assignment

Find the minimum cost assignment. Assignment of P₂ to J₂ and P₃ to J₁ is prohibited.

Cost Matrix (₹)

Person	J ₁	J ₂	J ₃
P ₁	4	6	8
P ₂	5	M	7
P ₃	M	6	4

M = very large cost
(e.g., 10^6)

Optimal Assignment : P₁ $\rightarrow$ J₁ (4), P₂ $\rightarrow$ J₂ (7), P₃ $\rightarrow$ J₃ (6)

Minimum total cost $Z = 4 + 7 + 6 = 17$

9. Key Points to Remember

- ✓ Assignment problem is a special case of transportation problem.
- ✓ Number of persons = number of jobs (add dummy if not equal).
- ✓ Use Hungarian method for optimal solution.
- ✓ For maximization, convert to minimization.
- ✓ For prohibited assignment, use a very large cost (M).
- ✓ Optimal assignment has exactly one assignment in each row and column.
- ✓ Total number of assignments = n .

10. Common GATE Questions

- ✓ Solve assignment problems (min/max).
- ✓ Problems with dummy row/column.
- ✓ Problems with prohibited assignments.
- ✓ Find total cost/profit of optimal assignment.
- ✓ Conceptual questions on Hungarian method.

11. Applications in Mining

- ✓ Assignment of machines to mining operations.
- ✓ Allocation of manpower to tasks.
- ✓ Assignment of equipment to sites.
- ✓ Job allocation in maintenance planning.
- ✓ Used in scheduling and resource allocation.



Network Analysis - Fundamentals

Network Terminology, Activities/Events, Network Construction, Paths, Critical Path Concept

1. What is Network Analysis?

- Network analysis is a project planning and control technique used to plan, schedule and monitor a set of interdependent activities.
- It represents the project using a network of activities and events.
- Helps in identifying the critical activities and project duration.

2. Basic Terminology

Term	Meaning
Activity	A task that takes time and resources to complete. (A, B, ...)
Event	A point in time indicating the start or completion of an activity.
Predecessor activity	An activity that must be completed before another can start.
Successor activity	An activity that follows another activity.
Immediate predecessor	An activity that directly precedes another.
Immediate successor	An activity that directly follows another.
Dummy activity	A fictitious activity with zero duration, used only to show precedence. (shown by dotted line)

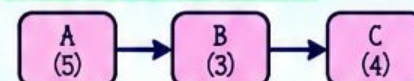
3. Types of Networks

(a) Activity on Arrow (AOA)



- Activities are represented by arrows.
- Events are represented by nodes (circles).

(b) Activity on Node (AON)



- Activities are represented by nodes (rectangles).
- Arrows show the precedence relationship.

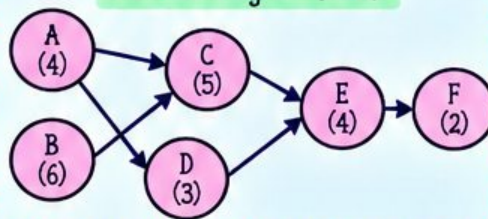
4. Rules for Network Construction

- Identify all activities and their precedence relationships.
- Draw the network maintaining logical sequence.
- No activity can start before its predecessor(s) are completed.
- Use dummy activity if required (in AOA).
- Each activity should be represented only once.
- The network must have a single start event and a single finish event.

5. Example - Network Construction

Activities : A(4), B(6), C(5), D(3), E(4), F(2)
Precedence: A, B → C
A → D
C, D → E
E → F

Network Diagram (AON)



6. Paths in a Network

- A path is a sequence of activities connecting the start event to the finish event.
- Path duration = Sum of durations of activities on that path.
- The longest path determines the project duration (may or may not be critical).

Paths for the above example:

- Path 1: A - C - E - F = 4 + 5 + 4 + 2 = 15
- Path 2: A - D - E - F = 4 + 3 + 4 + 2 = 13
- Path 3: B - C - E - F = 6 + 5 + 4 + 2 = 17

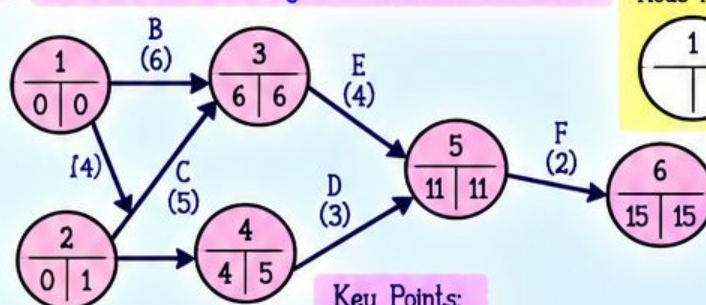
7. Critical Path Concept

- The critical path is the longest duration path in the network.
- Activities on the critical path have zero float (slack).
- Any delay in a critical activity delays the entire project.
- The project duration is equal to the duration of the critical path.

For the above example:

- Critical Path : **B - C - E - F**
- Project Duration = 17 units

8. Network with Early and Late Event Times



Node notation :



Key Points:

- Earliest time (TE) : Earliest occurrence of an event.
- Latest time (TL) : Latest occurrence of an event without delaying the project.
- Critical activities : TE = TL for both start and end events.

9. Example - Identify Critical Path

Activity	Predecessor	Duration
A	-	4
B	-	6
C	A, B	5
D	A	3
E	C, D	4
F	E	2

Paths:

- A - C - E - F = 15
- A - D - E - F = 13
- B - C - E - F = 17

Critical Path : **B - C - E - F**
Project Duration = 17

10. Important Formulas & Concepts

- Path duration = $\sum$ (activity durations).
- Critical path → longest duration path.
- Float (Slack) for an activity (i - j):
Total float = $TL(j) - TE(i) - t(i,j)$
- Critical activity → Total float = 0.
- Project duration = duration of critical path.

11. Applications in Mining

- Planning and scheduling of mining operations.
- Allocation of resources (men, machines, materials).
- Project planning for mine development (shafts, tunnels).
- Maintenance and shutdown planning.
- Helps in timely completion and cost control.



CPM & PERT

Critical Path Method, Earliest/Latest Times, Floats, Critical Activities, PERT Expected Time and Variance, Network Numericals

1. Critical Path Method (CPM) - Concept

- ➔ CPM is a deterministic network technique used for planning, scheduling and controlling projects.
- ➔ The critical path is the longest duration path in the network.
- ➔ Activities on the critical path have zero float (slack).
- ➔ Project duration = duration of critical path.
- ➔ Used when activity durations are known (deterministic).

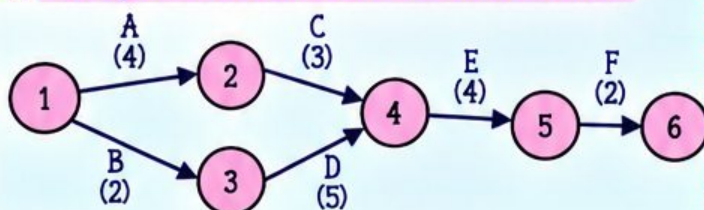
2. Basic Terminology

Term	Meaning
Activity	A task that consumes time and resources.
Event (Node)	A point in time (start or finish of activities).
Predecessor	An activity that must be completed before another starts.
Successor	An activity that follows another activity.
Critical Path	Longest duration path with zero float.
Float (Slack)	Amount of time an activity can be delayed without delaying the project.

3. Formulas in CPM

- Forward Pass (Earliest times)**
 $E_j = \max (E_i + t_{ij})$
 $E_1 = 0$
- Backward Pass (Latest times)**
 $L_i = \min (L_j - t_{ij})$
 $L_n = E_n$ (project duration)
- Total Float (TF)** : $TF = L_j - E_i - t_{ij}$
- Free Float (FF)** : $FF = E_j - E_i - t_{ij}$
- Independent Float (IF)** :
 $IF = \max (0, E_j - L_i - t_{ij})$

4. Example - CPM (Network and Calculations)



Act.	From	To	Duration t_{ij}	Earliest E_i	Earliest E_j	Latest L_i	Latest L_j	TF	FF
A	1	2	4	0	4	0	4	0	0
B	1	3	2	0	2	2	4	0	0
C	2	4	3	4	7	4	7	0	0
D	3	4	5	2	7	2	7	0	0
E	4	5	4	7	11	7	11	0	0
F	5	6	2	11	13	11	13	0	0

Critical Path : 1 - 2 - 4 - 5 - 6 (A - C - E - F)

Project Duration = 4 + 3 + 4 + 2 = 13 units

5. Program Evaluation and Review Technique (PERT)

- ➔ PERT is a probabilistic network technique used when activity durations are uncertain.
- ➔ Each activity has three time estimates :
 t_o : Optimistic time (shortest possible)
 t_m : Most likely time
 t_p : Pessimistic time (longest possible)

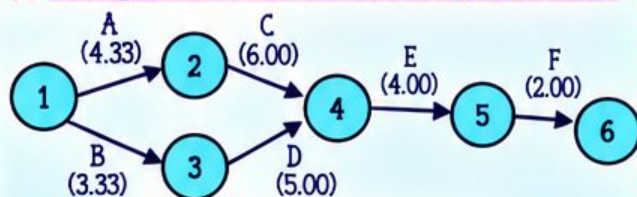
➔ Expected time (Mean) : $t_e = \frac{(t_o + 4t_m + t_p)}{6}$

➔ Variance : $\sigma^2 = \left(\frac{(t_p - t_o)}{6} \right)^2$; Standard deviation : $\sigma = \sqrt{\sigma^2}$

6. Example - PERT Calculations

Activity	t_o	t_m	t_p	Expected time $t_e = (t_o + 4t_m + t_p)/6$	Variance $\sigma^2 = ((t_p - t_o)/6)^2$
A	2	4	8	$(2 + 4(4) + 8)/6 = 26/6 = 4.33$	$(8 - 2)^2 / 36 = 1.00$
B	1	3	7	$(1 + 4(3) + 7)/6 = 20/6 = 3.33$	$(7 - 1)^2 / 36 = 1.00$
C	3	6	9	$(3 + 4(6) + 9)/6 = 36/6 = 6.00$	$(9 - 3)^2 / 36 = 1.00$
D	2	5	8	$(2 + 4(5) + 8)/6 = 30/6 = 5.00$	$(8 - 2)^2 / 36 = 1.00$
E	1	4	7	$(1 + 4(4) + 7)/6 = 24/6 = 4.00$	$(7 - 1)^2 / 36 = 1.00$

7. Example - PERT Network and Project Duration



Expected Project Duration (TE)
= 4.33 + 6.00 + 4.00 + 2.00
= 16.33 units

Variance of project duration (Critical path only) :
 $\sigma^2 = 1.00 + 1.00 + 1.00 + 1.00 = 4.00$ $\sigma = \sqrt{4} = 2.00$

8. Probability of Completing the Project

$$Z = \frac{T - T_E}{\sigma}$$

T = target completion time
 T_E = expected project duration
 σ = standard deviation of project duration

Example : Find probability of completing the project in 20 units.

$$Z = \frac{20 - 16.33}{2.00} = \frac{3.67}{2.00} = 1.835$$

From standard normal table,
 $P(Z \leq 1.835) \approx 0.9671$ (96.7%)

9. Key Points to Remember

- CPM uses deterministic times, PERT uses probabilistic times.
- Critical path has zero float.
- Project duration = sum of durations of activities on critical path.
- In PERT, $t_e = (t_o + 4t_m + t_p)/6$ and $\sigma^2 = ((t_p - t_o)/6)^2$.
- Probability of completion is found using standard normal distribution.

10. Common GATE Questions

- ✓ Find earliest and latest event times.
- ✓ Identify critical path and project duration.
- ✓ Calculate total, free and independent floats.
- ✓ Compute expected project duration and variance (PERT).
- ✓ Probability of completion within a given time.
- ✓ Network construction and interpretation.

11. Applications in Mining

- ✓ Planning and scheduling of mining projects.
- ✓ Scheduling of equipment installation and maintenance.
- ✓ Project planning for shaft sinking, tunneling and surface facilities.
- ✓ Resource allocation and manpower planning.
- ✓ Used to monitor and control project activities.



Inventory Models - Fundamentals

Inventory Concepts, Ordering Cost, Carrying Cost, Shortage Cost, EOQ, Assumptions and Basic Models

1. What is Inventory?

- Inventory refers to the stock of items held to meet future demand.
- It includes raw materials, work-in-progress, finished goods, spare parts, etc.
- Proper inventory management reduces total cost and ensures uninterrupted operations.

2. Types of Inventory

- Raw Material Inventory (basic materials for production)
- Work-in-Progress (WIP) (partially completed items)
- Finished Goods Inventory (completed products ready for sale)
- MRO Inventory (maintenance, repair and operating supplies)
- Safety Stock Inventory (extra stock to handle uncertainties)

3. Important Costs in Inventory Model

Cost Type	Symbol	Meaning	Examples
Ordering Cost (Setup Cost)	S or C_o	Cost incurred every time an order is placed	- Purchase order cost - Transportation cost - Inspection cost
Carrying Cost (Holding Cost)	H or C_h	Cost of holding one unit in inventory per unit time	- Storage cost - Insurance cost - Deterioration cost - Interest on capital
Shortage Cost (Stockout Cost)	π or C_s	Cost due to shortage or unmet demand	- Lost sales - Backorder cost - Customer dissatisfaction

4. Inventory Notations

- D = Demand rate (units/year)
 S = Ordering cost per order (₹/order)
 H = Holding cost per unit per year (₹/unit/year)
 C = Purchase cost per unit (₹/unit)
 Q = Order quantity (units)
 π = Shortage cost per unit per year (₹/unit/year)
 T = Ordering cycle time (years)
 I_{max} = Maximum inventory level
 I_{avg} = Average inventory level

5. Economic Order Quantity (EOQ) Model

- EOQ is the optimum order quantity which minimizes the total inventory cost (ordering cost + carrying cost).
- It assumes constant demand and instantaneous replenishment.

$$EOQ (Q^*) = \sqrt{\frac{2DS}{H}}$$

Total Annual Cost (TC)

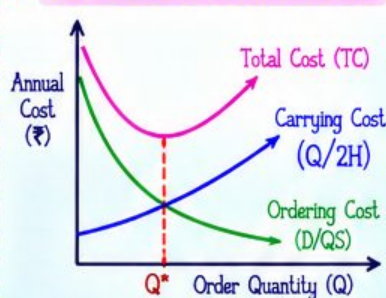
$$TC = \left(\frac{D}{Q}\right)S + \left(\frac{Q}{2}\right)H$$

At $Q = Q^* \rightarrow$ Minimum TC

Minimum Total Cost

$$TC_{min} = \sqrt{2DSH}$$

6. Cost Curves in EOQ Model



7. Assumptions of Basic EOQ Model

- Demand rate is constant and known.
- Lead time is zero or constant.
- Replenishment is instantaneous.
- Ordering cost per order is constant.
- Holding cost per unit per year is constant.
- No shortages are allowed.
- Item price is constant.
- Demand is independent (no seasonality).

8. Basic EOQ Model Characteristics

- Used for non-perishable items.
- Deterministic demand (known and constant).
- No shortages allowed.
- Focus is on minimizing the sum of ordering and carrying costs.
- Provides an optimal order quantity Q^* and minimum total cost.
- Widely used for raw materials and spare parts in mining and industrial operations.

9. Numerical Example (EOQ)

A mine requires 10,000 units of a spare part per year. Ordering cost per order is ₹500 and holding cost is ₹20 per unit per year. Find the EOQ and minimum total cost.

Given : D = 10,000 units/year
 S = ₹500/order
 H = ₹20/unit/year

Solution :

$$Q^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 10,000 \times 500}{20}} = \sqrt{500,000} = 707 \text{ units (approx)}$$

$$TC_{min} = \sqrt{2DSH} = \sqrt{2 \times 10,000 \times 500 \times 20} = \sqrt{200,000,000} = ₹14,142 \text{ (approx)}$$

10. Basic Inventory Models

Model	Assumptions	Application
EOQ Model (without shortages)	- Constant demand - No shortages - Instantaneous replenishment	Raw materials, spare parts
EOQ Model (with shortages)	- Shortages allowed - Includes shortage cost (π)	When backorders are permitted
Production Run (P-Q Model)	- Finite production rate - Production and consumption occur simultaneously	Manufacturing and mining production items
Quantity Discount Model	Unit price varies with order quantity	Bulk purchase scenarios

11. Key Takeaways

- Inventory models help to decide the optimal order quantity.
- EOQ minimizes the sum of ordering and carrying costs.
- Carrying cost increases with Q, ordering cost decreases with Q.
- Shortages lead to additional costs.
- Appropriate inventory model depends on the nature of demand, shortage policy and item type.

12. Applications in Mining

- Inventory of spare parts for heavy mining equipment.
- Storage of explosives, lubricants and consumables.
- Managing stock of replacement parts to avoid production stoppage.
- Useful in cost control and efficient resource planning.

Inventory Models - Numericals

EOQ Calculations, Reorder Level, Lead Time, Quantity Discounts, Shortages, Production/Inventory Models

1. EOQ Numerical Example

A company requires 10,000 units per year.
Ordering cost per order = ₹500.
Holding cost per unit per year = ₹20.
Find the Economic Order Quantity (EOQ) and minimum total annual cost.

Solution :

$$Q^* = \sqrt{\frac{2DS}{H}}$$

$$= \sqrt{\frac{2 \times 10,000 \times 500}{20}}$$

$$= \sqrt{500,000} = 707 \text{ units (approx)}$$

Minimum Total Cost (TC_{min}) :

$$TC_{min} = \sqrt{2DSH}$$

$$= \sqrt{2 \times 10,000 \times 500 \times 20}$$

$$= \sqrt{200,000,000}$$

$$= ₹14,142 \text{ (approx)}$$

2. Reorder Level (ROP)

Annual demand D = 12,000 units
Lead time L = 3 weeks
Demand per week = 300 units
Safety stock = 100 units
Find the Reorder Level.

Solution :

$$ROP = (\text{Demand during lead time}) + \text{Safety stock}$$

$$= (300 \times 3) + 100$$

$$= 900 + 100$$

$$= 1,000 \text{ units}$$

Formula :

$$ROP = d \times L + SS$$

d = demand rate per unit time
L = lead time
SS = safety stock

3. Lead Time and Safety Stock

Daily demand = 50 units
Lead time = 5 days
Maximum demand during lead time = 280
Find safety stock and reorder level.

Solution :

$$\text{Expected demand during LT} = 50 \times 5 = 250$$

$$\text{Safety Stock (SS)} = 280 - 250 = 30 \text{ units}$$

$$\text{Reorder Level (ROP)} = 250 + 30 = 280 \text{ units}$$

4. Quantity Discount Model

Annual demand D = 20,000 units
Ordering cost S = ₹400
Holding cost H = ₹10 per unit per year
Price schedule :

Price per unit (₹)	Applicable for order quantity
100	0 - 1,999
95	2,000 - 4,999
90	5,000 and above

Find the optimal order quantity and minimum total annual cost.

Solution :

Step 1 : Find EOQ for each price level

- For ₹100 : $Q^* = \sqrt{\frac{2 \times 20,000 \times 400}{10}} = 1,265 \text{ units}$
(Not feasible, since < 2,000)
- For ₹95 : $Q^* = 1,265 \text{ units}$ (feasible)
- For ₹90 : $Q^* = 1,265 \text{ units}$ (Not feasible since < 5,000)
So, check at 5,000 units

Step 2 : Calculate total cost for feasible options

Order Qty (Q)	Price (₹)	Ordering Cost (₹)	Holding Cost (₹)	Purchase Cost (₹)	Total Cost (₹)
2,000	95	4,000	9,500	19,00,000	19,13,500
1,265	95	6,325	6,013	19,00,000	19,12,338
5,000	90	1,600	25,000	18,00,000	18,26,600

Optimal order quantity = 5,000 units
Minimum total annual cost = ₹18,26,600

5. Inventory Model with Shortages

- Annual demand D = 10,000 units
 - Ordering cost S = ₹400
 - Holding cost H = ₹20 per unit per year
 - Shortage cost (penalty) P = ₹60 per unit per year
- Find the optimal order quantity, maximum inventory level and maximum shortage level.

Solution :

(1) Optimal order quantity (Q*) :

$$Q^* = \sqrt{\frac{2DS(H+P)}{HP}}$$

$$= \sqrt{\frac{2 \times 10,000 \times 400(20+60)}{(20 \times 60)}}$$

$$= \sqrt{106,667} = 327 \text{ units (approx)}$$

(2) Maximum inventory level (I_{max}) :

$$I_{max} = Q^* \times \frac{P}{(H+P)} = \frac{327 \times 60}{50/80} = 327 \text{ units (approx)}$$

(3) Maximum shortage level (S_{max}) :

$$S_{max} = Q^* \times \frac{H}{(H+P)} = \frac{327 \times 20}{80/80} = 327 \text{ units (approx)}$$

6. Production (Finite Production Rate) Model

Annual demand D = 10,000 units
Production rate P = 2,000 units/month
Demand rate d = 1,000 units/month
Setup cost S = ₹500
Holding cost H = ₹20 per unit per year
Find the optimal production quantity (EPQ).

Solution :

$$Q^* = \sqrt{\frac{2DS}{H(1 - \frac{d}{P})}}$$

$$= \sqrt{\frac{2 \times 10,000 \times 500}{20(1 - \frac{1,000}{2,000})}}$$

$$= \sqrt{\frac{500,000}{10}}$$

$$= \sqrt{50,000} = 224 \text{ units (approx)}$$

Formula :

$$Q^* = \sqrt{\frac{2DS}{H(1 - \frac{d}{P})}}$$

D = annual demand
S = setup cost
H = holding cost per unit per year
d = demand rate
P = production rate (P > d)

7. Sensitivity in EOQ

- ✓ If ordering cost (S) increases → EOQ increases.
- ✓ If holding cost (H) increases → EOQ decreases.
- ✓ If demand (D) increases → EOQ increases.

8. Key Formulas Summary

- EOQ : $Q^* = \sqrt{\frac{2DS}{H}}$
- Reorder Level : $ROP = dL + SS$
- Minimum Total Cost : $TC_{min} = \sqrt{2DSH}$
- EOQ with Shortages : $Q^* = \sqrt{\frac{2DS(H+P)}{HP}}$
- Production Model (EPQ) : $Q^* = \sqrt{\frac{2DS}{H(1 - \frac{d}{P})}}$

9. Important Assumptions

- ✓ Demand rate is constant and known.
- ✓ Lead time is constant.
- ✓ Ordering cost and holding cost are constant.
- ✓ Shortages are either allowed or not allowed based on the model.
- ✓ Production rate (P) is finite and greater than demand rate (P > d).
- ✓ No quantity deterioration (for basic models).

10. Applications in Mining

- ✓ Inventory of spare parts for heavy mining equipment.
- ✓ Stock of explosives, lubricants and consumables.
- ✓ Managing replacement parts in continuous mining operations.
- ✓ Useful in cost control and efficient resource planning.
- ✓ Determining optimal order quantity and reorder level.
- ✓ Handling shortages in remote mining sites.

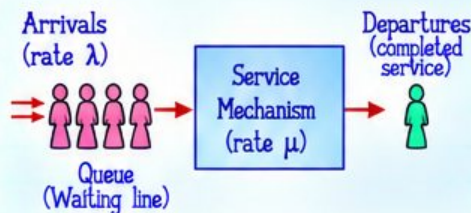
Queuing Theory - Fundamentals

Queue Terminology, Arrival Rate, Service Rate, Waiting Line Characteristics, Kendall Notation, Queue Disciplines

1. What is Queuing Theory?

- Queuing theory studies the waiting lines formed when the demand for a service is more than the available service capacity.
- It is used to analyze and design systems with waiting lines.
- Common applications : banks, toll booths, hospitals, repair shops, call centers, mining equipment maintenance, etc.

2. Basic Queue Model

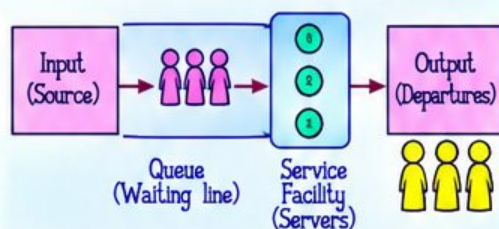


- ✓ Customers arrive → wait in queue → receive service → leave the system.
- ✓ Arrival rate (λ) : average number of arrivals per unit time.
- ✓ Service rate (μ) : average number of customers served per unit time.
- ✓ Queue length : number of customers waiting.
- ✓ Waiting time : time spent in queue or in system.

3. Key Terminology

Term	Meaning
Customer	Entity requiring service (person, machine, job, etc.)
Arrival rate (λ)	Average number of arrivals per unit time (e.g., per hour)
Service rate (μ)	Average number of customers served per unit time
Service time	Time required to serve one customer ($1/\mu$)
Interarrival time	Time between two successive arrivals ($1/\lambda$)
Waiting line (Queue)	Customers waiting for service
Queue length (L_q)	Average number of customers in queue
System length (L)	Average number of customers in system (queue + service)
Waiting time (W_q)	Average time spent in queue
System time (W)	Average time in system (waiting + service)

4. Queueing System Structure



- ✓ Input source : finite or infinite population.
- ✓ Queue : waiting line of customers.
- ✓ Service facility : one or more servers.
- ✓ Output : customers leaving after service.
- ✓ System capacity : maximum number allowed in the system (may be infinite).

5. Kendall Notation

Kendall notation : $A/B/c/K/N/D$

Symbol	Meaning
A	Interarrival time distribution
B	Service time distribution
c	Number of servers (service channels)
K	System capacity (maximum number in the system)
N	Population size (calling population)
D	Queue discipline

Examples :

- $M/M/1$: Poisson arrivals, exponential service, 1 server, infinite capacity, infinite population, FIFO.
- $M/M/c$: Poisson arrivals, exponential service, c servers.
- $M/D/1$: Poisson arrivals, deterministic service time, 1 server.

6. Common Distributions

(a) **M** : Markovian (Poisson arrivals / exponential service times)

- ✓ Interarrival time and service time are exponentially distributed.
- ✓ Memoryless property.
- ✓ Most widely used in practice.

(b) **D** : Deterministic

- ✓ Constant interarrival or service time.

(c) **G** : General

- ✓ Any arbitrary distribution.

Examples :

- $M/M/1$: Poisson arrivals, exponential service, 1 server.
- $M/M/c$: Multiple servers.
- $M/D/1$: Poisson arrivals, deterministic service time.
- $G/G/1$: General distributions.

7. Queue Disciplines (Service Rules)

- FIFO (First In First Out)**
Customers are served in the order of arrival (most common).
- LIFO (Last In First Out)**
Last arrived customer is served first (stack mechanism).
- SIRO (Service In Random Order)**
Customers are served in a random order.
- Priority Queue**
Customers with higher priority are served first (e.g., emergency cases).
- Preemptive / Non-preemptive Priority**
In preemptive, ongoing service can be interrupted by higher priority customer.

8. Basic Queue Characteristics

Symbol	Meaning
λ	Arrival rate (customers/unit time)
μ	Service rate (customers/unit time)
$\rho = \lambda/\mu$	Utilization factor (server busy fraction)
L_q	Average number in queue
L	Average number in system (queue + service)
W_q	Average waiting time in queue
W	Average time in system
P_0	Probability of zero customers in the system
P_n	Probability of n customers in the system

For a stable system (steady state) : $\rho = \frac{\lambda}{\mu} < 1$
(i.e., arrival rate < service rate)

9. Applications in Mining

- ✓ Queuing analysis for mine equipment maintenance facilities.
- ✓ Waiting time of dumpers/trucks at loading/unloading points.
- ✓ Bottleneck analysis in material handling systems.
- ✓ Design of service facilities such as workshops and repair stations.
- ✓ Helps in optimal allocation of service resources (machines, manpower).

10. Key Takeaways

- ✓ Queuing theory models waiting lines in service systems.
- ✓ Kendall notation helps in identifying the type of queue.
- ✓ Arrival rate (λ) and service rate (μ) are fundamental parameters.
- ✓ Queue discipline affects performance.
- ✓ Widely used in mining operations and industrial systems.

Queuing Models & Numericals

M/M/1, Utilization, L, L_q, W, W_q, Little's Law, Multi-server Basics, GATE-level Problems

1. M/M/1 Queue Model

- ➔ M/M/1 means :
Markovian arrivals,
Markovian service times,
Single server.
- ➔ Arrivals follow Poisson
distribution with rate λ
(customers/unit time).
- ➔ Service times are
exponentially distributed
with rate μ
(customers/unit time).
- ➔ Queue discipline : FIFO
(usually).
- ➔ Assumptions : infinite
population, infinite
queue capacity,
FCFS, steady state
($\lambda < \mu$).

2. M/M/1 Formulas ($\lambda < \mu$)

Parameter	Formula
Utilization (ρ)	$\rho = \frac{\lambda}{\mu} \quad (\rho < 1)$
Average number in system (L)	$L = \frac{\lambda}{\mu - \lambda}$
Average number in queue (L _q)	$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$
Average time in system (W)	$W = \frac{1}{\mu - \lambda}$
Average waiting time in queue (W _q)	$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$
Little's Law	$L = \lambda W \text{ and } L_q = \lambda W_q$

3. Key Relationships

- ✓ $\rho = \lambda/\mu$: fraction of time server is busy.
- ✓ L : average number of customers
in the system (queue + service).
- ✓ L_q : average number of customers
in the queue only.
- ✓ W : average time a customer spends
in the system.
- ✓ W_q : average time a customer waits
in the queue.
- ✓ Little's Law : $L = \lambda W$ and $L_q = \lambda W_q$
(valid for all stable systems).

4. Numerical Example - M/M/1

A mine equipment repair shop receives
on average 6 machines per hour ($\lambda = 6$).
The service rate is 8 machines per hour
($\mu = 8$). Find ρ , L, L_q, W and W_q.

Solution :

- Utilization : $\rho = \frac{\lambda}{\mu} = \frac{6}{8} = 0.75$
- $L = \frac{\lambda}{\mu - \lambda} = \frac{6}{8 - 6} = 3$ machines
- $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{6^2}{8(8 - 6)} = \frac{36}{16} = 2.25$ machines
- $W = \frac{1}{\mu - \lambda} = \frac{1}{8 - 6} = 0.5$ hours
- $W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{6}{8(8 - 6)} = \frac{6}{16} = 0.375$ hours

5. Interpretation of Results

- ✓ On average 3 machines are in the
system (2.25 waiting + 0.75 in service).
- ✓ A machine waits 0.375 hours (22.5 min).
in the queue.
- ✓ Total time in system is 0.5 hours (30 min).
- ✓ Server is busy 75% of the time.

6. M/M/s Queue Model (Multi-server)

- ✓ M/M/s : Markovian arrivals, Markovian
service times, s identical servers.
- ✓ Arrivals : Poisson (rate λ)
- ✓ Service times : Exponential (rate μ)
- ✓ Traffic intensity : $\rho = \frac{\lambda}{s\mu} \quad (\rho < 1)$
- ✓ $P_0 = \left[\sum_{n=0}^{s-1} \frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{(\frac{\lambda}{\mu})^s}{s!(1-\rho)} \right]^{-1}$
- ✓ $L_q = P_0 \left(\frac{\lambda}{\mu} \right)^s \frac{\rho}{s!(1-\rho)^2}$
- ✓ $L = L_q + \frac{\lambda}{\mu}$
- ✓ $W_q = \frac{L_q}{\lambda}$, $W = W_q + \frac{1}{\mu}$

7. Numerical Example - M/M/s

A mine workshop has 3 identical service
servers. Arrivals are 12 machines per hour
($\lambda = 12$). Service rate per server is 5 machines
per hour ($\mu = 5$). Find ρ , P₀, L_q, W_q and L.

Solution :

- Traffic intensity : $\rho = \frac{\lambda}{s\mu} = \frac{12}{3 \times 5} = 0.8$
- $P_0 = \left[1 + \left(\frac{\lambda}{\mu} \right) + \frac{(\frac{\lambda}{\mu})^2}{2!} + \frac{(\frac{\lambda}{\mu})^3}{3!(1-\rho)} \right]^{-1}$
 $= \left[1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{6(0.2)} \right]^{-1}$
 $= 0.0666$ (approx)
- $L_q = 0.0666 \times \frac{2.4^3 \times 0.8}{6(0.2)^2} = 5.12$ machines
- $W_q = \frac{L_q}{\lambda} = \frac{5.12}{12} = 0.427$ hours (25.6 min)
- $L = L_q + \frac{\lambda}{\mu} = 5.12 + \frac{12}{5} = 7.52$ machines

8. GATE-Level Problem

In an M/M/1 queue, the average time in
the system is 10 minutes and the average
waiting time in the queue is 5 minutes.
Find the arrival rate (λ) and service rate (μ).

Solution :

Given : $W = 10 \text{ min} = \frac{1}{6} \text{ hr}$, $W_q = 5 \text{ min} = \frac{1}{12} \text{ hr}$

$$1) \text{ From } W = \frac{1}{\mu - \lambda} \Rightarrow \mu - \lambda = 6 \quad \dots(1)$$

$$2) \text{ From } W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{1}{12}$$

$$\Rightarrow \frac{\lambda}{6\mu} = \frac{1}{12} \Rightarrow \lambda = \frac{\mu}{2} \quad \dots(2)$$

3) Substitute (2) in (1) :

$$\mu - \frac{\mu}{2} = 6 \Rightarrow \frac{\mu}{2} = 6$$

$$\Rightarrow \mu = 12 \text{ customers/hr}$$

$$\Rightarrow \lambda = \frac{\mu}{2} = 6 \text{ customers/hr}$$

9. Important Points

- ✓ For stable system : $\rho < 1$ (i.e., $\lambda < \mu$
for M/M/1 and $\lambda < s\mu$ for M/M/s).
- ✓ As $\rho \rightarrow 1$, L, L_q, W, W_q increase rapidly.
- ✓ Little's Law holds for all queueing models.
- ✓ Multi-server system reduces waiting time
and queue length.
- ✓ Queueing models are useful in mines for
equipment repair shops, loading/unloading
facilities, haul trucks, etc.

10. Key Takeaways

- ✓ Understand M/M/1 and M/M/s models.
- ✓ Be comfortable with utilization, L, L_q, W, W_q.
- ✓ Use Little's Law for quick calculations.
- ✓ Check stability condition ($\rho < 1$).
- ✓ Practice GATE-level numericals with
interpretation of results.
- ✓ Apply queueing models to real mining
operations (trucks, shovels, repair shops).

Reliability + LP + Transportation Practice

Mixed Numerical Problems Covering Reliability, Availability, LP, Transportation and Assignment

1. Reliability Numerical Problem

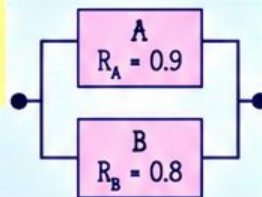
Q: A system consists of two components in parallel. Component A has reliability 0.9 and component B has reliability 0.8. Find the reliability of the system.

Solution :

For two components in parallel :

$$\begin{aligned} R_p &= 1 - (1 - R_A)(1 - R_B) \\ R_p &= 1 - (1 - 0.9)(1 - 0.8) \\ &= 1 - (0.1)(0.2) \\ &= 1 - 0.02 = 0.98 \end{aligned}$$

System Reliability = 0.98



2. Availability Numerical Problem

Q: A machine has MTBF = 100 hours and MTTR = 10 hours. Find its steady-state availability.

Solution :

$$\text{Availability (A)} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

$$A = \frac{100}{100 + 10} = \frac{100}{110} = 0.909$$

Availability = 0.909 (90.9%)

Key Formulas :

- $A = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$
- Unavailability = $1 - A$
- Reliability (constant failure rate) : $R(t) = e^{-\lambda t}$
- $\text{MTBF} = \frac{1}{\lambda}$

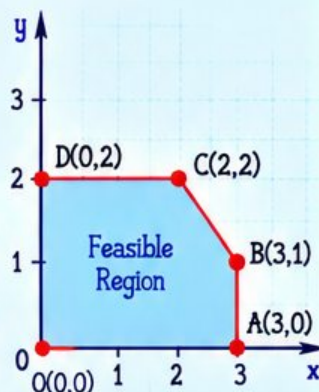
3. Linear Programming Numerical Problem

Q: Maximize $Z = 3x + 2y$
Subject to $x + y \leq 4$
 $x \leq 3$
 $y \leq 2$
 $x, y \geq 0$

Solution : Corner Points

Point	x	y	$Z = 3x + 2y$
O	0	0	0
A	3	0	9
B	3	1	11
C	2	2	10
D	0	2	4

Maximum $Z = 11$ at $(x, y) = (3, 1)$



4. Transportation Numerical Problem

Q: Solve the following transportation problem to minimize total cost.

Source	Cost (₹ per unit)			Supply
	D1	D2	D3	
S1	2	3	1	20
S2	5	4	8	30
Demand	10	25	15	50

Solution : Using Least Cost Method (Initial BFS)

Allocation (units)	D1	D2	D3	Supply
S1	0	5	15	20
S2	10	20	0	30
Demand	10	25	15	50

$$\begin{aligned} \text{Total Cost} &= (5 \times 3) + (15 \times 1) + (10 \times 5) + (20 \times 4) \\ &= 15 + 15 + 50 + 80 = 160 \end{aligned}$$

Total Transportation Cost = ₹160

5. Assignment Numerical Problem

Q: Assign 4 jobs to 4 machines with minimum total cost.

Job	Cost Matrix (₹)			
	M1	M2	M3	M4
J1	0	2	7	8
J2	6	4	3	7
J3	5	8	1	8
J4	7	6	9	4

Solution : (Using Hungarian Method - Final Assignment)

Job	Assigned Machine	Cost
J1	M2	2
J2	M3	3
J3	M1	5
J4	M4	4

Minimum Total Cost = $2 + 3 + 5 + 4 = 14$

Steps (Hungarian Method) :

- 1 Row reduction (subtract row minimum).
- 2 Column reduction (subtract column minimum).
- 3 Cover all zeros with minimum number of lines.
- 4 Make adjustments if required.
- 5 Select independent zeros and assign jobs.

6. Mixed Practice Questions (GATE Level)

- 1 Reliability : Three components with reliabilities 0.9, 0.8 and 0.7 are in series. Find system reliability.
- 2 Availability : A system has MTBF = 200 hours and MTTR = 20 hours. Find availability and unavailability.
- 3 Linear Programming : Minimize $Z = 4x + 3y$
Subject to $2x + y \geq 6$, $x + 2y \geq 6$, $x, y \geq 0$.
Find the optimal solution using graphical method.
- 4 Transportation : Solve the following using least cost method and find the total cost.

Source	D1	D2	D3	Supply
S1	4	6	8	40
S2	5	4	7	60
Demand	30	50	20	100

- 5 Assignment : Find the minimum total cost for the following.

Job	M1	M2	M3	M4
J1	10	14	8	9
J2	9	7	12	6
J3	8	8	6	7
J4	7	9	10	5

7. Key Takeaways

- ✓ Reliability and availability are important for maintaining system performance.
- ✓ LP helps in optimal decision making under constraints.
- ✓ Transportation and assignment problems are special cases of LP.
- ✓ Practice mixed numericals to strengthen problem-solving skills.
- ✓ Focus on concepts, formulas and steps for accurate solutions in GATE.

8. Important Formulas (Quick Revision)

- ✓ Reliability (series) : $R_s = \prod R_i$
- ✓ Reliability (parallel) : $R_p = 1 - \prod (1 - R_i)$
- ✓ Availability : $A = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$
- ✓ Little's Law : $L = \lambda W$, $L_q = \lambda W_q$
- ✓ Transportation : Minimize $Z = \sum_{i,j} c_{ij} x_{ij}$
- ✓ Assignment : Minimize $Z = \sum_{i,j} c_{ij} x_{ij}$ (one-to-one)

9. Practice Tip

- ✓ Attempt at least 10 mixed numerical problems.
- ✓ Check units and feasibility.
- ✓ Draw graphs for LP problems.
- ✓ Use systematic steps for transportation and assignment.
- ✓ Verify results with alternative methods where possible.

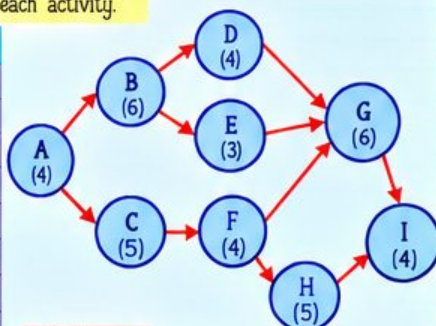
Network + Inventory + Queuing Practice

CPM/PERT, EOQ/Inventory and Queuing Numerical Problems + Important Formulas

1. Network (CPM) Numerical Problem

Q: Activities, precedence and duration (days) are given below. Find the critical path, project duration and total float for each activity.

Activity	Predecessor	Duration
A	—	4
B	A	6
C	A	5
D	B	4
E	B	3
F	C	4
G	D, E	6
H	F	5
I	G, H	4



Solution :

Critical Path : A - B - D - G - I
Project Duration = 24 days

2. Network (PERT) Numerical Problem

Q: For the following activities, time estimates (in days) are given. Find the expected project duration and variance of the critical path.

Activity	Predecessor	a	m	b
A	—	2	4	6
B	A	3	6	9
C	A	1	4	7
D	B	2	5	8
E	C	3	6	9
F	D, E	2	4	6

Solution (on Critical Path A - B - D - F) :

Activity	t_e	Variance (σ^2)
A	4	0.44
B	6	1.00
D	5	1.00
F	4	0.44

Expected Project Duration :

TE = 4 + 6 + 5 + 4 = 19 days

Project Variance :

$$\sigma_p^2 = 0.44 + 1 + 1 + 0.44 = 2.88 \Rightarrow \sigma_p = \sqrt{2.88} = 1.70$$

PERT Formulas :

• Expected time : $t_e = (a + 4m + b) / 6$

• Variance : $\sigma^2 = \left(\frac{b - a}{6} \right)^2$

• Project variance : $\sigma_p^2 = \sum \sigma^2 \text{ (on CP)}$
 $\sigma_p = \sqrt{\sigma_p^2}$

3. Inventory (EOQ) Numerical Problem

Q: A mine requires 12,000 units of a spare part per year. Ordering cost per order = ₹400 and holding cost = ₹20 per unit per year. Find (i) EOQ (ii) Number of orders per year (iii) Time between orders (iv) Minimum annual cost.

Solution :

1) EOQ (Q^*) :

$$Q^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 12,000 \times 400}{20}} = \sqrt{480,000} = 693 \text{ units (approx)}$$

2) Number of orders : $N = \frac{D}{Q^*} = \frac{12,000}{693} = 17.3 \approx 17$

3) Time between orders : $T = \frac{365}{N} = \frac{365}{17.3} = 21 \text{ days}$

4) Minimum annual cost :

$$TC_{\min} = \sqrt{2DSH} = \sqrt{2 \times 12,000 \times 400 \times 20} = \sqrt{192,000,000} = ₹13,856 \text{ (approx)}$$

4. Inventory with Shortages Numerical

Q: Annual demand = 16,000 units, ordering cost = ₹500/order, holding cost = ₹25/unit/year, shortage cost = ₹75/unit/year. Find optimal order quantity (Q^*), maximum inventory level ($I_{\max}$), and maximum shortage level ($S_{\max}$).

Key Formulas :

$$Q^* = \sqrt{\frac{2DS(H+P)}{HP}}$$

$$I_{\max} = \left(\frac{P}{H+P} \right) Q^*$$

$$S_{\max} = \left(\frac{H}{H+P} \right) Q^*$$

Solution :

$$Q^* = \sqrt{\frac{2 \times 16,000 \times 500 (25 + 75)}{25 \times 75}} = \sqrt{853,333} = 924 \text{ units (approx)}$$

$$I_{\max} = \frac{75}{(25 + 75)} \times 924 = 693 \text{ units}$$

$$S_{\max} = \frac{25}{(25 + 75)} \times 924 = 231 \text{ units}$$

5. Quantity Discount Inventory Numerical

Q: Demand = 20,000 units/yr, Ordering cost = ₹400, Holding cost = ₹10/unit/yr. Price schedule :

Price (₹/unit)	Quantity Range
100	0 - 1,999
95	2,000 - 4,999
90	5,000 and above

Find the optimal order quantity and minimum annual cost.

Solution :

Price	EOQ (units)	Relevant Q	Total Annual Cost (₹)
100	1,265	1,265	2,00,000 + 5,060 = 2,05,060
95	1,265	2,000	1,90,000 + 4,000 = 1,94,000
90	1,265	5,000	1,80,000 + 2,000 = 1,82,000

Optimal order quantity = 5,000 units

Minimum Annual Cost = ₹1,82,000

6. Queuing (M/M/1) Numerical Problem

Q: A loading facility receives trucks at an average rate of 6 per hour. The service rate is 8 per hour. Find ρ , L , L_q , W and W_q .

Solution :

$$\rho = \frac{\lambda}{\mu} = \frac{6}{8} = 0.75$$

$$L = \frac{\lambda}{\mu - \lambda} = \frac{6}{8 - 6} = 3 \text{ trucks}$$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{6^2}{8(8 - 6)} = 2.25 \text{ trucks}$$

$$W = \frac{1}{\mu - \lambda} = \frac{1}{2} = 0.5 \text{ hr (30 min)}$$

$$W_q = \frac{L_q}{\lambda} = \frac{2.25}{6} = 0.375 \text{ hr (22.5 min)}$$

7. Queuing (M/M/s) Numerical Problem

Q: Arrivals = 12 customers/hour, Service rate = 5 customers/hour per server, Number of servers = 3. Find ρ , P_0 , L_q , W_q and L .

Solution :

$$\rho = \frac{\lambda}{s\mu} = \frac{12}{3 \times 5} = 0.8$$

$$P_0 = \left[\sum_{n=0}^{s-1} \left(\frac{\lambda^n}{n!} \right) + \frac{\left(\frac{\lambda^s}{s!} \right)}{s!(1 - \rho)} \right]^{-1}$$

$$= \left[1 + 2.4 + \frac{2.4^2}{2!} + \frac{2.4^3}{s!(1 - 0.8)} \right]^{-1}$$

$$= 0.0666 \text{ (approx)}$$

$$L_q = P_0 \frac{\lambda^s \rho}{s!(1 - \rho)^2} = 5.12 \text{ customers}$$

$$W_q = \frac{L_q}{\lambda} = \frac{5.12}{12} = 0.427 \text{ hr (25.6 min)}$$

$$L = L_q + \frac{\lambda}{\mu} = 5.12 + 2.4 = 7.52 \text{ customers}$$

8. Key Formulas Summary

(A) Network (CPM)

- ES, EF, LS, LF, Total Float = LS - ES
- Critical Path : zero total float
- Project Duration : sum of activity times on CP

(B) Network (PERT)

- $t_e = \frac{(a + 4m + b)}{6}$ $\sigma^2 = \left(\frac{b - a}{6} \right)^2$
- Project variance = $\sum \sigma^2 \text{ (on CP)}$, $\sigma_p = \sqrt{\sigma_p^2}$

(C) Inventory

- EOQ : $Q^* = \sqrt{\frac{2DS}{H}}$
- With shortages : $Q^* = \sqrt{\frac{2DS(H+P)}{HP}}$
- $I_{\max} = (P/(H+P)) Q^*$, $S_{\max} = (H/(H+P)) Q^*$

(D) Queuing

- M/M/1 : $\rho = \frac{\lambda}{\mu}$, $L = \frac{\lambda}{\mu - \lambda}$, $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$
 $W = \frac{1}{\mu - \lambda}$, $W_q = \frac{L_q}{\lambda}$
- M/M/s : Use standard formulas with P_0 , L_q , W_q , L

Formula + Fact Sheet & Master Revision

All Systems Engineering Formulas, Shortcuts, Model Assumptions, Comparison Tables and Common GATE Traps

1. Linear Programming (LP)

Standard Form : $\text{Max/Min } Z = c^T x$
Subject to $Ax \leq / \geq / = b, x \geq 0$

Graphical Method (2 variable) :

Find corner points & evaluate Z.

Simplex Method : Best for large problems.

Important Points :

- ✓ Optimal solution lies at a corner point.
- ✓ Add slack/surplus variables.
- ✓ Artificial variable method (Big-M / Two-Phase) for $\geq$ or $=$ constraints.
- ✓ Multiple optimal solutions if Z is same at adjacent corner points.

2. Transportation Model

Formulation :

Minimize $Z = \sum_i \sum_j c_{ij} x_{ij}$

Subject to :

$$\sum_j x_{ij} = \text{supply}_i \quad (i = 1, 2, \dots, m)$$

$$\sum_i x_{ij} = \text{demand}_j \quad (j = 1, 2, \dots, n)$$

$$x_{ij} \geq 0$$

Methods :

- North West Corner (Initial BFS)
- Least Cost Method (Initial BFS)
- Vogel's Approximation Method (Initial BFS)
- MODI / Stepping Stone (Optimality Test)

Key Points :

- If $\sum \text{supply} \neq \sum \text{demand} \rightarrow$ add dummy row/column with zero cost.
- Number of basic variables = $m + n - 1$.

3. Assignment Model

Minimize : $Z = \sum_i \sum_j c_{ij} x_{ij}$

Subject to :

$$\sum_j x_{ij} = 1 \quad (i = 1, 2, \dots, n)$$

$$\sum_i x_{ij} = 1 \quad (j = 1, 2, \dots, n)$$

$$x_{ij} = 0 \text{ or } 1$$

Hungarian Method (Steps) :

- 1 Row reduction (subtract row minimum)
- 2 Column reduction (subtract column minimum)
- 3 Cover all zeros with minimum lines
- 4 If lines = $n \rightarrow$ optimal solution
- 5 If lines $< n \rightarrow$ adjust matrix
- 6 Assign jobs (independent zeros)

4. Network Analysis (CPM)

Forward Pass :

Earliest Start (ES) = max (EF of predecessors)

Earliest Finish (EF) = ES + t

Backward Pass :

Latest Finish (LF) = min (LS of successors)

Latest Start (LS) = LF - t

Total Float (TF) = LS - ES = LF - EF

Free Float (FF) = min (ES of successors) - EF

Critical Path : path with zero total float

Project Duration : Sum of activity times on critical path

5. Network Analysis (PERT)

Three Time Estimates :

$$t_e = \frac{a + 4m + b}{6}$$

Variance : $\sigma^2 = \left(\frac{b - a}{6}\right)^2$

Project Variance : $\sigma_p^2 = \sum (\text{on CP}) \sigma^2$

Project S.D : $\sigma_p = \sqrt{\sigma_p^2}$

Probability : $Z = \frac{T - \mu_p}{\sigma_p}$

6. Inventory Models

(1) Basic EOQ : $Q^* = \sqrt{\frac{2DS}{H}}$
 $TC_{\min} = \sqrt{2DSH}$

(2) With Shortages :

$$Q^* = \sqrt{\frac{2DS(H + P)}{HP}}$$

$$I_{\max} = \frac{P}{H + P} \cdot Q^*$$

$$S_{\max} = \frac{H}{H + P} \cdot Q^*$$

(3) Production (EPQ) :

$$Q^* = \sqrt{\frac{2DS}{H(1 - \frac{d}{p})}}$$

(4) Reorder Level : $R = d \times L + SS$

(5) Safety Stock : $SS = Z \sigma_L$

7. Queuing Models

M/M/1 Formulas :

$$\rho = \frac{\lambda}{\mu} \quad (\rho < 1)$$

$$L = \frac{\lambda}{\mu - \lambda}$$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$W = \frac{1}{\mu - \lambda}$$

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

Little's Law : $L = \lambda W, L_q = \lambda W_q$

M/M/s Formulas :

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{(\frac{\lambda}{\mu})^s}{s!(1 - \rho)} \right]^{-1}$$

$$L_q = P_0 \left(\frac{\lambda}{\mu} \right)^s \frac{s}{s!(1 - \rho)^2}$$

$$L = L_q + \frac{\lambda}{\mu}, W = \frac{L}{\lambda}, W_q = \frac{L_q}{\lambda}$$

8. Reliability & Availability

Series System : $R_s = \prod R_i$

Parallel System : $R_p = 1 - \prod (1 - R_i)$

Constant Failure Rate : $R(t) = e^{-\lambda t}$

$$MTBF = \frac{1}{\lambda}, MTTF = \frac{1}{\lambda}$$

Availability (steady state) :

$$A = \frac{MTBF}{MTBF + MTTR}$$

Unavailability : $Q = 1 - A$

For Two Components in Parallel :

$$R_p = 1 - (1 - R_1)(1 - R_2) = R_1 + R_2 - R_1 R_2$$

9. Model Assumptions (Summary)

Model	Key Assumptions
LP	Linearity, Additivity, Divisibility, Certainty, Non-negativity
Transportation	Single homogeneous product, Known supply & demand, Constant unit cost, No capacity constraints
Assignment	Square matrix, One-to-one assignment, Known fixed costs
CPM	Deterministic activity times, No resource constraints, Well-defined precedence
PERT	Probabilistic activity times (a, m, b), Beta distribution, Independent activities
Inventory	Constant demand, Instantaneous replenishment, Known costs, No deterioration (basic EOQ)
Queuing	Poisson arrivals, Exponential service times, FIFO (usually), Infinite population and queue capacity
Reliability	Independent components, Constant failure rate (for exponential), Repairable or non-repairable as specified

10. Comparison Tables

CPM vs PERT		
Aspect	CPM	PERT
Activity time	Deterministic	Probabilistic
Time estimates	Single	Three (a, m, b)
Primary use	Time-cost tradeoff	Project planning under uncertainty
Critical path	Unique	May vary
Variance	Not considered	Considered

M/M/1 vs M/M/s		
Aspect	M/M/1	M/M/s
Servers	1	s (≥ 1)
Traffic intensity	$\rho = \lambda/\mu$	$\rho = \lambda/(s\mu)$
Formulas	Simple	More complex
Waiting time	Higher	Lower
Use case	Small systems	Large systems

Inventory Models		
Model	When to use	Key Formula
Basic EOQ	No shortages	$Q^* = \sqrt{2DS/H}$
EOQ with shortages	Shortages allowed	$Q^* = \sqrt{\frac{2DS(H+P)}{HP}}$
Production (EPQ)	Finite production rate	$Q^* = \sqrt{\frac{2DS}{H(1 - \frac{d}{p})}}$

11. Common GATE Traps

- 1 Wrong preprocessing steps in Hungarian method.
- 2 For transportation, forgetting dummy row/column when unbalanced.
- 3 Misinterpreting float definitions (TF vs FF).
- 4 In PERT, using CPM formula instead of expected time formula.
- 5 Using population variance instead of project variance in PERT.
- 6 Taking $\rho \geq 1$ in queuing systems (unstable).
- 7 Confusing Little's Law formulas.
- 8 Using EOQ formula in shortage/production cases.
- 9 Ignoring units (hours, days, rates).
- 10 Assuming components are dependent in reliability problems.
- 11 Calculation errors in exponentials and square roots.
- 12 Not checking feasibility of LP solution.

12. Quick Revision Checklist

- ✓ Memorize all key formulas.
- ✓ Understand model assumptions.
- ✓ Practice at least 5-10 numericals per topic.
- ✓ Be able to identify the correct model quickly.
- ✓ Revise comparison tables and GATE traps.
- ✓ Focus on units and interpretation.
- ✓ Attempt previous GATE questions.

Full Systems Engineering Mock Test

Mixed MCQ + MSQ + NAT | Reliability, LP, Transportation, Assignment, Network, Inventory & Queuing

1. Reliability (MCQ)

A system has three components in series with reliabilities 0.9, 0.8 and 0.7. The system reliability is :

- (A) 0.504
- (B) 0.720
- (C) 0.846
- (D) 0.940

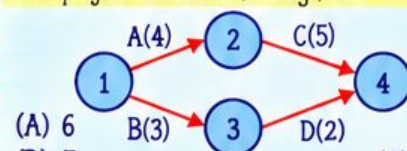
2. Reliability (NAT)

A component has constant failure rate $\lambda = 0.002$ per hour. Find the reliability of the component for 500 hours. (Answer up to 3 decimal places)

$$R(t) = e^{-\lambda t}$$

3. Network Analysis (CPM) (MCQ)

The figure shows an activity network. The project duration (in days) is :



- (A) 6
- (B) 7
- (C) 9
- (C) 8
- (D) 9

4. Network Analysis (PERT) (NAT)

An activity has three time estimates $a = 4$, $m = 6$, $b = 16$ (days). Find (i) Expected time (t_e) and (ii) Variance (σ^2).

$$t_e = \frac{a + 4m + b}{6} \quad \sigma^2 = \left(\frac{b - a}{6} \right)^2$$

5. Linear Programming (MCQ)

Maximize $Z = 5x + 4y$
Subject to : $2x + y \leq 10$
 $x + 2y \leq 8$
 $x, y \geq 0$

The optimal value of Z is :

- (A) 20
- (B) 24
- (C) 25
- (D) 28

6. Linear Programming (MSQ)

Which of the following are assumptions of a Linear Programming model? (More than one option may be correct)

- (A) Linearity in objective and constraints
- (B) Divisibility of variables
- (C) Certainty of coefficients
- (D) Integer decision variables
- (E) Non-negativity of decision variables

7. Transportation Problem (MCQ)

A balanced transportation problem has 3 sources and 4 destinations. The basic feasible solution will have _____ number of basic variables.

- (A) 6
- (B) 7
- (C) 9
- (D) 12

8. Transportation Problem (NAT)

The following table gives the transportation costs (₹/unit) and supply/demand.

Source	D1	D2	D3	Supply
S1	4	6	8	20
S2	5	4	7	30
Demand	25	15	10	

Using Least Cost Method, find the total transportation cost.

9. Assignment Problem (MSQ)

In the assignment problem, which of the following are correct? (More than one option)

- (A) Number of assignments = number of jobs
- (B) Objective is to minimize total cost/time
- (C) Hungarian method provides optimal solution
- (D) Number of independent zeros = $n - 1$ for optimality
- (E) Problem remains same if a constant is added to all costs.

10. Inventory Model (MCQ)

Annual demand $D = 12,000$ units, Ordering cost $S = ₹400$ per order, Holding cost $H = ₹20$ per unit per year. The Economic Order Quantity (EOQ) is :

- (A) 300
- (B) 400
- (C) 490
- (D) 600

11. Inventory with Shortages (NAT)

Annual demand = 10,000 units, Ordering cost = ₹500 per order, Holding cost = ₹20 per unit per year, Shortage cost = ₹50 per unit per year. Find the optimal order quantity (Q^*), maximum inventory (I_{max}) and maximum shortage (S_{max}).

12. Inventory (MSQ)

Which of the following are assumptions of the basic EOQ model? (More than one option may be correct)

- (A) Constant demand rate
- (B) Known and constant ordering cost
- (C) Constant unit purchase cost
- (D) No shortages allowed
- (E) Infinite planning horizon.

13. Queuing Theory (MCQ)

In an M/M/1 queue, $\lambda = 6$ customers/hr and $\mu = 8$ customers/hr. The average number of customers in the system (L) is :

- (A) 1.5
- (B) 3.0
- (C) 4.0
- (D) 6.0

14. Queuing Theory (NAT)

An M/M/s system has arrival rate $\lambda = 12$ customers/hr, service rate $\mu = 5$ customers/hr per server and $s = 3$ servers. Find (i) utilization (ρ) and (ii) probability of zero customers (P_0). (Answer up to 3 decimal places)

15. Queuing Theory (MSQ)

Which of the following statements about queuing systems are correct? (More than one option may be correct)

- (A) Little's Law : $L = \lambda W$ is valid for all stable systems
- (B) M/M/1 is a single server model
- (C) In M/M/s, increasing number of servers always decreases waiting time
- (D) L_q represents average number of customers in queue
- (E) Queue discipline has no effect on performance

16. Comprehensive (MCQ)

Which of the following techniques is most suitable to minimize the total project duration while considering resource constraints?

- (A) Linear Programming
- (B) Transportation Model
- (C) CPM with crashing
- (D) EOQ model

17. Mixed Concepts (NAT)

A project has three activities in series with durations 5, 7 and 9 days. Each activity can be crashed by 2 days at a cost of ₹1,000, ₹1,500 and ₹2,000 per day respectively. If the project has to be completed in 15 days, find the minimum crashing cost.

18. Comprehensive (MSQ)

Which of the following statements are correct regarding Systems Engineering models? (More than one option may be correct)

- (A) Reliability models are useful for maintenance planning
- (B) Inventing models consider uncertainty in demand as standard
- (C) Queuing models help in analyzing waiting lines
- (D) LP can handle integer decision variables directly
- (E) Transportation and assignment are special cases of LP.